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ABSTRACT

This is a narrative review of research in mathematics education reported during 1987. The purpose of this review is to extract from research reports ideas that may prove useful to school practitioners. Major sections are: (1) "Planning for Instruction" (relating historical developments, aides and grades, teaching approaches, problem solving, drill practice, mental computations, and attitudinal factors); (2) "Mathematical Content and Materials"; (3) "Individual Differences, Evaluation, and Learning Theory"; (4) "Teacher Education" (containing research on preservice and inservice teacher education); (5) "College Level Instruction" (considering prominent researchers and teachers, content, learning, prediction of success, word problems, student errors, remediation, computers, anxiety and sex differences); (6) "Research Summaries"; and (7) "Epilogue: Recommendations for Future Research" (identifying 11 problem areas). A total of 288 references are listed. (YP)

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A REVIEW AND SYNTHESIS
OF
RESEARCH IN MATHEMATICS EDUCATION
REPORTED DURING
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PREFACE

This is a narrative review of research in mathematics education reported during 1987. It is intended primarily for school practitioners: teachers, supervisors, principals, and superintendents. It is hoped that it will also be useful to college professors and researchers in mathematics education. As is true of most narrative reviews, the purpose of this review is to extract from research reports ideas that may prove useful to practitioners. There is no claim that the ideas which are highlighted throughout the volume represent stable results that enjoy a high degree of certainty. If you like an idea, try it with your classes, and, perhaps, even research that idea in greater detail. It is through the collective judgments of practitioners that "conventional wisdom" grows and hopefully improves mathematics education.

This is not a novel! One should not read it from the first page to the last, but rather one should approach the work with an encyclopedic attitude. This can be done in at least two ways. One approach is to read the Introduction, scan the Table of Contents, and select the pages that interest you; and a second approach is to thumb through the pages, reading the ideas in rectangles until you find one that fascinates you, then read about the research related to the idea. For those who wish to dig deeper, the list of references at the end of the volume will provide further information.

This review is an outgrowth of the compilation of research studies completed by Marilyn Suydam and reported in the July 1988 issue of the Journal for Research in Mathematics Education. Suydam included 20 research summaries, 165 articles, and 290 dissertations in that compilation. She also provided short annotations for those reports which were felt to be most germane to mathematics education in North America. These annotated items became the primary focus of this review, but, in addition, other documents in the ERIC system, primarily research reports given at professional meetings, were reviewed.

The review is analytic, synthetic, and summative. An attempt was made to analyze the research reports, synthesize their primary findings, and provide a summative statement that would capture the essence of each report. Throughout the volume, ideas that might be particularly useful to practitioners are highlighted by enclosing them in rectangles.

Many deserve thanks and deep appreciation: the hundreds of researchers who spent countless hours in their research and writing; the reviewers and editors who helped fashion the final

reports of these researchers; Robert W. Howe and the staff at the ERIC Clearinghouse for Science, Mathematics, and Environmental Education who provided copies of the reports; Marilyn Suydam who gave invaluable editorial advice and assistance; John Dossey and an anonymous reviewer from OERI who made insightful suggestions for improvement of the volume; Janie Blackburn who patiently typed and edited the entire manuscript; and the countless school practitioners who provided the initial impetus for this report.

Donald J. Dessart

February, 1989
Knoxville, Tennessee

As you have noted from the title, this publication summarizes the research in mathematics education for one year, 1987. In that year, 165 articles describing research studies were published, plus 290 dissertations, 19 research summaries on individual mathematical topics, and several dozen ERIC documents. These numbers are cited so that you become aware that developing this summary was no easy task.

To prepare this summary, we turned to a leading mathematics educator with a great deal of experience in this field. Donald J. Dessart has had a distinguished career of working with preservice and inservice teacher education programs. He has had wide experience in interpreting research findings for the classroom teacher, including the publication of several interpretive summaries for the National Council of Teachers of Mathematics. And among other facets of his background is his service at the National Science Foundation, where, among other things, he considered indicators of good mathematics instruction.

Dr. Dessart is uniquely qualified to prepare this summary not only by his background, but by his careful, thorough approach to such a task. He not only sifted through the masses of words, but culled the studies for their meaning and their usefulness to teachers. It is apparent that he brings his own teaching experience to this effort, highlighting points of particular interest to teachers.

We are very pleased that he agreed to prepare this summary -- and hope that you, the reader, will find his analysis not only interesting but useful.

We would also like to thank a very busy person who reviewed this manuscript for us: John A. Dossey, Professor of Mathematics at Illinois State University, and Past President of the National Council of Teachers of Mathematics. His perspective on research has been of immense help.

Marilyn N. Suydam
Associate Director
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INTRODUCTION

This review of research in mathematics education reported in 1987 consists of seven major sections: I, Planning for Instruction; II, Mathematical Content and Materials; III, Individual Differences, Evaluation, and Learning Theory; IV, Teacher Education; V, College-Level Instruction; VI, Research Summaries; and VII, Epilogue: Recommendations for Future Research.

In Section I, Planning for Instruction, we view research related to six broad topics: historical developments; aides and grades; teaching approaches; problem solving; drill, practice, and mental computations; and attitudinal factors. Historical research helps us to improve the present by avoiding the errors of the past. Teacher aides are becoming more prevalent in mathematics classrooms, but we must learn their most effective contributions. Decisions regarding many approaches to teaching must be made before the teacher enters the classroom. Problem solving is an attitude or frame of mind which must be adopted early, and it must be conveyed to students. Drill, practice, and mental computations have always been controversial practices, but most teachers feel that conventional wisdom demands their use. Many teachers agree that building favorable attitudes toward mathematics is far more important than achievement in mathematics.

Section II, Mathematical Content and Materials, deals with research related to the structure and organization of mathematical content, the various methods for teaching this content, and the instructional materials used to convey the content to students. The tools of the teacher are obviously important ingredients in the instructional process and warrant study and research.

Individual differences, evaluation, and learning theory are the primary topics of Section III. Differences due to achievement, disabilities, mathematical anxiety, and gender must be dealt with in the mathematics classroom. Evaluation is crucial to progress but yet is a highly sensitive issue needing much research. Theories of learning hopefully guide the teaching enterprise in mathematics but vary so greatly that teachers are often left confused as to the proper directions to pursue in teaching.

Section IV, Teacher Education, consists of two main parts: preservice teacher education and inservice teacher education. Research on preservice teacher education studied perceptions of prospective teachers about mathematics and teaching, the preparation of preservice teachers in mathematics and pedagogy, computers and preservice teachers, and student teaching. Research on inservice

teacher education consisted of studies of opinions of teachers about mathematics and the teaching of mathematics and effective models for inservice teacher education.

In Section V, College-Level Instruction, nine major research topics are considered. These include prominent teachers and researchers, content related to learning, methods of teaching and learning, prediction of success, studies of word problems, studies of student errors, remediation of mathematical difficulties, college students and computers, and mathematical anxiety and gender differences. Most of the research was done on problems related to the first two years of college rather than later years.

Section VI, Research Summaries, consists of five sections: an annual comprehensive research listing, Investigations in Mathematics Education, topical narrative reviews, single topic reviews for practitioners, and meta-analytic and best-evidence syntheses. Summaries of research are often very difficult endeavors but are necessary to prevent the field from wandering aimlessly. The meta-analytic and best-evidence syntheses represent efforts to introduce statistical rigor into the review process.

Section VII, the concluding section, is entitled "Epilogue: Recommendations for Future Research." In this section, the author of this report attempts to identify eleven problem areas for future research efforts. These eleven areas cover most of the popular and current areas of research interest.

Finally, the reader is urged to read the Preface for suggestions of how to read and study this report of research in mathematics education reported during 1987.

I. PLANNING FOR INSTRUCTION

Many hours of work and effort take place before the teacher enters the classroom. This planning by supervisors, teachers, teacher-aides, parents, and students is absolutely essential to the success of the teaching enterprise. Consequently, research on topics related to "planning for instruction" is wanted and needed by the mathematics education community.

In this section on "Planning for Instruction" of this review and synthesis of research in mathematics education reported during 1987, we shall look at six broad areas of research: historical developments; aides and grades; teaching approaches; problem solving; drill, practice, and mental computation; and attitudinal factors. Each of these topics should be the subject of thought and planning before the teacher enters the classroom, and research on these topics should aid the teacher in coping with the day-to-day realities of teaching mathematics.

Historical study helps us to understand the present by understanding the past and more importantly to insure that we may avoid the pitfalls and mistakes of the past. Teacher aides are becoming increasingly important to mathematics teachers and especially to teachers of less-talented students. Teaching approaches, whether geometrical or ways to educate foreign-born students, are important. Problem solving is the central focus of attention of many, many mathematics educators. A document without a discussion of problem solving is seriously lacking. Drill, practice, and mental computation have been the interests of researchers in mathematics education for decades. Although research interest has diminished somewhat in recent years, drill and practice still are crucial topics for planning. Finally, attitudes of students toward mathematics, computers, and themselves determine the success or failure of students in ways that are often unrelated to mathematical abilities.

Historical Developments

Taking an historical view in research on mathematics and mathematics education always seems to produce interesting and revealing conclusions. Harper (February, 1987) examined children's conceptual development of certain algebraic ideas related to their historical development. McConnel (April, 1987) studied the evolution of objectives in secondary school teaching of mathematics during the period of 1918-82. DuPre (January, 1987) and Nibbelink, Stockdale, Hoover, and Mangru (September, 1987) reviewed the modern mathematics movement during the passage of some 25 years.

Algebraic Concepts. Should mathematical concepts be taught to parallel their phylogenetic evolution, that is, the way in which the mathematical concept evolved in history? Mathematics educators have debated or at least discussed this notion from time to time. Harper looked at the historical evolution of algebraic concepts and pointed out that they have passed through three stages: rhetorical, syncopated, and symbolic. Rhetorical algebra is the algebra of the period before Diophantus (about 250 AD), when algebraic arguments were written in longhand and symbols were not used; syncopated algebra extended from the time of Diophantus through the end of the sixteenth century, during which time letters came to represent unknown quantities; and finally symbolic algebra, introduced by Vieta (about 1580), used letters to represent "given quantities"; e.g., the use of "a" in $x + y = a$ as opposed to the use of "7" in $x + y = 7$.

Harper presented a problem similar to the following to 144 British secondary grammar school students from years 1 through 6, who were ranked by ability:

> Problem: Suppose the sum of two numbers is 100 and their difference 40. What are the two numbers?

He observed students solving the problem, in which they were allowed as much time as they wanted. The methods used by the students were classified as "Rhetorical" if the student did not use algebraic symbolism to solve the problem, e.g., divide the sum by 2, the difference by 2, and add these two to get one of the numbers; "Diophantine" if the pupil used a symbol to represent an unknown quantity, e.g., $x + y = 100$, $x - y = 40$, $2x = 140$, $x = 70$; and finally "Vietan" if the student introduced n as the sum of x and y and m as the difference of x and y, producing the general equations, $x + y = n$ and $x - y = m$, so that $x = \frac{m + n}{2}$.

The frequencies of the use of the three methods varied from Rhetorical: 4, Diophantine: 0, and Vietan: 0 in year 1 (about 11 years old) to Rhetorical: 0, Diophantine: 4, and Vietan: 20 in year 6 (about 16 years old), with the Rhetorical approach prevalent in the early years and the Vietan method more prevalent in the later years.

Harper noted that the step from the Diophantine to the Vietan system took 1300 years to evolve, whereas, the change takes place in less than five years in the classroom. It is also suggested that, perhaps, the introduction of symbols into classroom algebra might

be delayed until the Vietan case is needed. Most mathematics educators would probably find this practice difficult to accomplish in the classroom.

Objectives in Secondary School Mathematics. McConnel (April, 1987) identified and classified objectives for teaching secondary school mathematics in the United States from 1918-1982 as reflected in statements of objectives found in the periodical literature. The statements were classified into Knowledge, Process, Attitude and Interest, or Cultural Awareness categories. Surprisingly, statements in the Cultural Awareness category were most numerous, followed by those in the Process Category. Also, the objectives rated as most important in the literature came from the Cultural Awareness and Process categories.

Modern Mathematics. The modern mathematics era has been subjected to rather severe criticism in recent years. DuPre (January, 1987) investigated the validity of these criticisms as related to the University of Illinois Committee on School Mathematics (UICSM) materials. The purpose of UICSM, originated in the 1950s, was to prepare secondary school materials so that students entering college could begin immediately the study of calculus and analytic geometry.

DuPre summarized the criticisms of the new mathematics into nine questions which were then sent to former UICSM staff members. Not surprisingly the responses were quite favorable to UICSM. DuPre concluded that UICSM did not deserve much of the criticism that was directed toward modern mathematics programs and that the UICSM program, if taught in the way it had been intended, would have produced better achievement than did the traditional curriculum. This latter conclusion is subject to much speculation, but one cannot dispute the finding that modern mathematics programs did stimulate greater involvement of mathematical specialists in curricular reform movements.

Modern mathematics has been subject to much criticism, some valid and some invalid; however, no one can dispute that it stimulated the interest of many mathematical specialists in curriculum reform.

One of the interesting analyses of the modern mathematics movement in relation to other periods of mathematical development

was provided in Nibbelink, Stockdale, Hoover, and Mangru (September, 1987). They studied the period from 1955 to 1985 in relation to textbook practices and achievement trends in problem solving in the elementary grades. They partitioned the 30-year period into four broad periods: (a) Old Math (the late 1950s); (b) New Math (mid 60s and early 70s); (c) Aftermath (the late 70s); and (d) New Old Math (the early and late 80s).

They analyzed the three textbook series that had been most widely used in Iowa during each of the three periods. In particular, they classified problems in those textbooks into the categories: (a) multiple step, (b) clue word given, and (c) like prior problem. One might surmise that this also represents a difficulty ordering of the problems from most to least difficult. During the New Math period, the percentage of multiple-step problems dropped to 13 percent of the total, from 27 percent in the late 1950s. Following the New Math period, the percentages of multiple-step problems gradually rose until they reached 20 percent in the late 1980s.

Over the same 30-year period, the Iowa Testing Programs maintained a method for making absolute achievement comparisons over various subtests. In particular, if one traces achievement patterns in problem solving for grades 4, 5, and 6 during the 30-year period, some common patterns are found. Mathematical problem solving reached its highest point in about 1965 and its lowest point in 1975, with gradual gains occurring in the 1980s.

The authors noted that during the Old Math era it was a common practice to introduce new concepts with a story problem, whereas, in the New Math period, concepts and definitions were given heavy emphasis with a deemphasis upon problem solving. The Aftermath era (late 70s) stressed "back to basics" with concern for computation, which was followed by a heavy emphasis upon problem solving in the 1980s. These trends appear to parallel the achievement patterns in problem solving discussed earlier.

Achievement in problem solving of students in grades 4, 5, and 6 dropped in Iowa during the years of the New Mathematics (1965-1975), probably due to less emphasis in classrooms upon problem solving.

Aides and Grades

Teacher aides are becoming more prevalent and useful in the classroom. Palmer (March, 1987) explored the usage of student teachers as aides in middle school mathematics classes. Middle school students were also the subject of a study by MacIver (October, 1987), who investigated the influence of various classroom and student characteristics upon students' self-assessments of their mathematical abilities.

Aides. Palmer organized two pre-algebra classes and three remedial general mathematics classes with student teachers as aides. There were also four remedial general mathematics classes that were taught by two-teacher teams. All classes were observed using a coding system based upon process-products. The students and teachers were also interviewed and field data were collected to establish trends and themes.

The remedial general mathematics classes with student teacher aides exhibited more students on task, fewer off-task behaviors, very few students waiting for assistance, more time on entire class teaching, and a brisk lesson pace, when compared with remedial general mathematics classes with only the teachers. Consequently, the use of student aides represented very positive features for these classes. The classes with two teachers were also influenced positively with high-level interactive lessons. However, not surprisingly, student teachers as aides appeared to produce very few differences in the pre-algebra classes.

Grades. MacIver reported that the literature suggests that upper elementary school pupils are strongly inclined to feel that they are mathematically competent. One may wonder why students of varying abilities may reach such a conclusion. In particular, do classroom tasks, grading practices, or grouping procedures influence these positive self evaluations?

After analyzing data on 1,570 upper elementary students (primarily fifth and sixth graders), MacIver concluded that the combination of differentiated task structures (e.g., students are provided choices of various class lessons) and infrequent grades was associated with the students' low reliance upon adult assessments (e.g., what the teacher thinks of me as a student) and across-domain comparisons (e.g., how well do I do in mathematics compared to other subjects) for their positive self-concepts in mathematics. But the combination of differentiated task structure and frequent grades was associated with heavy reliance upon adult assessments for the positive self concept. Since grades are often a product of adult assessments of children and frequently given grades reinforces this fact with children, it is understandable

that children would rely upon grades for positive self-concepts. When grades are given infrequently, less talented students look to selective social comparisons (e.g., how well do I do in mathematics in comparison to other children) and exaggerated estimates of their mastery of mathematical tasks for their positive self-concepts.

Teaching Approaches

As the teacher of any grade level approaches the teaching assignment, he or she must be concerned with many factors that will influence his or her students and classes. In the United States, we are continuing to have students enter our schools who had their earlier schooling in other lands and cultures. Quite naturally the teacher thinks and plans according to the logic of his or her own culture; but can this logic differ drastically from the logic of other cultures?

The Whorf Hypothesis. Zepp (February, 1987) discussed the Whorf Hypothesis. This hypothesis contends that a student's thinking and logical processes are dependent on the student's first language. Consequently, when students attempt thinking and reasoning in a second language, they experience great difficulty and probably return to the first language to carry out the thought processes. One might speculate that if the great mathematicians of the past had lived in cultures different from the ones that they did that the field of mathematics might have grown quite differently than it has.

The Whorf Hypothesis contends that one's thinking and logical thought processes are dependent on his or her first language. Does this possibility create learning difficulties for students new to our culture?

Zepp administered a logic test consisting of implicational and disjunctive sentences in English to first-language English students, in Chinese to first-language Chinese students, and in English to Chinese students of an English language university. No differences were found in their ability to make implications. Differences among the groups on the disjunctive sentences were attributed to poor command of and low confidence in English. Logic and language are vital factors, but grouping practices and types of instruction also affect learning.

Paired Learning and Direct Instruction. Delgado (December, 1987) compared paired learning with individual learning for 166 fifth-grade Mexican-American students learning division in four schools in San Jose, California. It was hoped that students working in pairs would have improved attitudes toward mathematics, would exhibit greater time on task, and would achieve higher than students working alone. A regression analysis revealed no significant differences between the paired and the working alone treatments.

Cruz de Nason (May, 1987) looked at the effects of teaching 12 relational concepts to first-grade students in schools of low-income families in Puerto Rico. A Spanish version of the Boehm Test of Basic Concepts was given as a pretest and a posttest to two randomly assigned groups of children: the Instructed Group (75 students) and the Traditional Group (50 students). The Instructed Group received 32 lessons on 12 concepts missed most frequently on the pretest. The Traditional Group received non-systematic instruction. The results revealed that the systematic instruction had beneficial effects on concept acquisition, with differences between the two groups being greatest on the targeted concepts.

Keogh (January, 1987) studied the effectiveness of self-instructional and didactic instructional methods with both normal first graders and mildly mentally retarded youngsters. The results suggested that the self-instructional method was superior to the didactic instructional method in learning mathematical skills, particularly with low ability and mentally retarded children.

Traditional and Transformational Geometry. Bradley (October, 1987) compared the effects on student achievement of a traditional and a transformational approach to teaching geometry. Two groups of eleventh-grade students in a suburban college preparatory school learned geometry by each of the two methods. The two groups were shown to be equivalent in aptitude and knowledge of geometry before the study began.

For five and one-half months, one group received traditional geometrical instruction, whereas, the other group received geometrical instruction based upon transformations (reflections, translations, rotations, and dilations). Posttesting of the two groups revealed that both had achieved adequately in geometrical understandings, with the transformational class showing greater gains. Surprisingly, the transformational class also showed a greater gain in its understanding of formal geometry than the traditional group.

Problem Solving

The current, broad interest in problem solving among mathematics educators was reflected in the large number of research studies

completed in 1987 in the area of problem solving. The studies can be partitioned into three broad categories: (a) heuristics or strategies that are used in solving problems; (b) factors that may stimulate problem-solving behaviors; and (c) the evaluation and testing of problem solving. Each of these broad areas of problem-solving research is important, but, perhaps, the last, the evaluation and testing of problem solving, is the most crucial to future progress of research in this area. We will look at research related to each of the three broad categories.

Heuristics of Problem Solving. Clinical interview is a technique used in describing heuristics used by children in problem solving. This technique was used by Flexer (November, 1987) and De Corte and Verschaffel (November, 1987) in studying problem solving in first-grade children.

Flexer conducted clinical interviews with two high-ability first graders on alternate weeks over a full academic year. Problems in computation, logic, number theory, and geometry were presented to the two children, one male and one female, to observe how they solved the problems. Computation problems consisted of the addition of two numbers less than five; addition and subtraction problems of one, two, or three-digit numbers were constructed only from the digits one through five. The novel three-shape addition problem in which the child is given the three sums of three combinations of three numbers (arranged in three circles as sums of chips in the circles), taken two at a time, and then finds the number of chips in each circle, was also presented to the children. Two- and three-piece square geometrical puzzles were given in which the children were requested to construct triangles, squares, trapezoids, or rectangles.

The male, Jesse, certified by the school district as gifted, searched for rules to apply and could apply them quite well. The female, Kirsten, preferred to develop her own methods for solving problems. Jesse would search for the correct answer, whereas, Kirsten seemed to enjoy the process of solving the problems. Jesse's style was described as extrinsic, while Kirsten's was classified as intrinsic.

Furthermore, Jesse would tend to persist in a procedure that he thought was correct but was very willing to change any answer that was produced by the procedure. Kirsten, on the other hand, was reluctant to change an answer that she felt was derived from a trusted method. Jesse was quick to abandon a problem, if the method of solution was not obvious, whereas, Kirsten would persist and was reluctant to give up.

Flexer hypothesized that the intrinsic problem solver would be the more productive problem solver when compared to the extrinsic

problem solver. She felt that her observations supported this assumption. She further noted that if this assumption is correct then teachers should delay offering rules, reinforcing answers, and emphasizing algorithms, and instead stress personal methods of solving problems. She also observed that the conventional method of identifying gifted students by test scores seemed to identify the poorer problem solver in this one case.

De Corte and Verschaffel (November, 1987) collected data on the problem representations and solution strategies of 30 first graders in three interviews over an academic year. The children were given a series of simple addition and subtraction word problems. During each interview, the children were given eight word problems, four involving addition and four in subtraction. During the interviews, a set of 15 blue and 15 green cubes were made available to the children as well as two cardboard puppets, named Pete and Ann. The children were advised that they could use any of these objects to help solve the problems.

The authors concluded that the children used a large variety of strategies to solve problems and that these became internalized over time. The children's strategies for solving the word problems appeared to depend highly upon the semantic structure of the problem as well as the sequence of numbers in the problem. The investigators had asked each child to represent the solution of the problem with blocks and puppets. This action may have intensified the importance of the problem structure with the children.

The investigators seriously questioned the current practice of presenting word problems to children only as an application of addition and subtraction algorithms. They suggested that such word problems should become more often the vehicle for meaningful instruction of the actual algorithms of addition and subtraction.

Using word problems only as <u>applications</u> of learned algorithms of addition and subtraction may not be most productive. Perhaps, word problems should become the actual vehicle for initially learning the concepts and algorithms of these processes.

Writt (July, 1987) explored the effects that strategy had on problem solving and heuristics and especially upon Polya's four phases of problem solving: Understand the Problem, Devise a Plan,

Carry out the Plan, and Look Back. He distinguished strategy as a problem specific procedure which will produce a correct answer and heuristic as a non-problem specific procedure which does not necessarily produce a correct result. To carry out the study, a computer program, The Square Problem, was devised. The Square Problem could be solved by four strategies and 12 heuristics, classified by experts as belonging to one of Polya's four phases. Seventy-five New York City high school juniors and seniors solved the problem four different times.

It was found that most of the differences among the strategies employed by the students occurred in the Polya phase of Carry Out the Plan. The unsuccessful problem solvers spent more time on Understand the Problem and Carry Out the Plan than the successful problem solvers. It was significant that none of the students used the Polya phase of Look Back.

Germain-McCarthy (April, 1987) concentrated upon studying the heuristic, "Think of a Simpler Problem" (TSP). Problems suitable to TSP were identified and categorized according to why, where, and when TSP might be employed. Seven self-instructional booklets were constructed for the treatment students, who were above average New York City high school students. Pre- and posttests were devised to measure the students' abilities to apply TSP to an entire problem. The results of the study indicated a significant improvement in the treated groups and no significant differences in the untreated groups.

Stonecipher (March, 1987) explored the mathematical problem-solving processes of ten gifted and ten average junior high school students. The gifted students used inductive reasoning and routine checks quite regularly, whereas, the average students used trial-and-error methods more often. The gifted students reasoned inductively on numerous occasions, and the average students frequently misinterpreted the problems.

First-year algebra students were studied by Rebovich (May, 1987) and Kamal (July, 1987). Rebovich looked at processes and strategies employed by algebra students in solving word problems, and Kamal compared successful and less successful strategies of students in writing equations for word problems. Rebovich had the students solve three number, three motion, and three mixture problems. The results indicated that students had developed a consistent repertoire for solving number problems, but they did not employ consistent schema for solving motion and mixture problems. The students had little difficulty solving equations derived from the problems. Kamal, on the other hand, examined the processes in writing equations for simple and complex mixture problems. He

found that successful and less successful students employed standard procedures for writing equations for simpler problems, but only the successful students seemed to be aware of the concepts involved. When solving complex problems, the successful students monitored their work so as to see the need for transforming parts of the problems. The unsuccessful students either did not see the need for such monitoring or simply were unable to do it.

In addition to researching the problem process, researchers of 1987 also studied the elements that students remembered for solving problems in the future. Chukwu (January, 1987) and Foss (June, 1987) investigated this retention phenomena. Chukwu attempted to determine the extent to which eighth- and ninth-grade students would continue to use heuristics over a two- and four-week period after continuous heuristic instruction. The results revealed that there was a definite pattern in the use of heuristics after instruction ceased. Foss concentrated upon studying how deriving solutions to problems might enhance the memory of these problems and their solutions for future use. She found a clear superiority for retention of self-derived solutions when compared to solutions which were studied from a textbook.

Memory of problem-solving processes for future use by students is important. Research seems to indicate that self-generated problem solving processes are retained longer by students than those processes which are read or copied. Try to encourage independent problem solving!

Factors Stimulating Problem Solving. Since problem solving has become the focus of attention of so many mathematics educators, research should naturally turn to a study of the factors that may affect problem solving. These factors range from belief systems of students to the order of the elements in problem presentations. Hard, concrete evidence concerning these factors is hard to find, but many interesting and potentially useful results have appeared.

Buchanan (November, 1987) hoped to describe differences in problem-solving performance of third-grade gifted and average fifth-grade students during an eight-week period. She assumed that there would be differences between the two groups. Specifically, she hypothesized that the gifted students would use novel approaches, demonstrate greater insight and intuition, be more motivated, and have more positive attitudes than the average students. Surprisingly, Buchanan's conclusions were unrelated to the gifted/average labels

she had attached to the two groups and were more related to attitudes, motivations, and belief systems of the total group.

After cautioning the reader that qualitative, observational research is subject to bias due to the participation of the researcher, Buchanan noted that the most significant differences were found between girls and boys. The girls appeared to be more ego-involved and tended to pursue a problem to gain teacher or peer approval rather than the satisfaction of solving the problem, itself. Throughout the investigative period, girls approached problems more cooperatively and took longer to solve than boys. The boys, on the other hand, became more task-committed and were less ego-involved than the girls. Buchanan raised the question as to what prior experiences might have contributed to these differences in the problem-solving behaviors of girls and boys.

Anand and Ross (March, 1987) used computer-assisted instruction to personalize arithmetic materials for fifth- and sixth-grade children. In one version, examples were personalized by incorporating information about the friends, interests, and hobbies of the students, and in the other two versions, concrete and abstract contexts were used. The results favored the personalized materials over the other two contexts in solving standard and transfer problems, in recognizing rule procedures, and in task attitudes.

Anand and Ross offered several explanations for the seeming superiority of the personalized materials over the concrete and abstract materials. These reasons were: (a) increased interest by the students in the problem-solving task; (b) increased internal logic and organization of the personalized materials over the other two kinds; (c) the association of a mathematical rule with a meaningful, integrated set of ideas (e.g., three pizzas cut in fourths will give me 12 pieces to share with my friends); and (d) reduced comprehension problems for students.

The transfer of problem-solving methods acquired on one occasion to the solution of a different set of problems was the subject of study by Zollman (February, 1987) and Cooper and Sweller (December, 1987). Zollman noted that the question of the order of problem presentation (more difficult-less difficult or less difficult-more difficult) to maximize positive transfer of problem-solving learning had been equivocal. He decided to control the order of problem presentation but to vary the inner-structure of the problem (the number of variables, the number of conditions, or the size of the solution). These variances did not lead to any measurable transfer of learning for either order of presentation (more to less difficult or less to more difficult). Cooper and Sweller also conducted a series of experiments on algebra problems,

finding that students who had been subjected to a heavy emphasis upon worked examples or an extended acquisition period were better able to solve transfer problems than students who had only been subjected to conventional materials.

Hasty (March, 1987) claimed that when students translate a statement such as, "There are six times as many students as professors at this university" into an equation, the majority of them incorrectly write 6S = P rather than correctly write 6P = S. She felt that the current methods for teaching translation of statements into equations did not adequately handle this problem. She designed a 45-minute lesson to remedy this situation. She tested the lesson with two seventh-grade classes, two high school classes, and two university classes. For the high school and university classes, the 45-minute lesson resulted in a significant reduction of reversals from pretests to retention tests.

Have your algebra students write an equation to represent, "There are six times as many students as professors." Many may incorrectly write, 6S = P, which indicates greater care is needed in translation problems.

Firsching (January, 1987) examined classroom textbooks, recorded classroom sessions, and interviewed students and teachers in order to correlate instructional patterns with student performance on word problems. He found that the primary technique of presentation, i.e., implicit, inductive teaching by example, caused students to reach inferences in problem solving without necessarily helping them to reach correct inferences.

The effects of verbalization was the topic of an investigation by Roark (March, 1987). Seventy-three students ranging in age from 14 through 16 years worked on the Tower of Hanoi Puzzle with three and four disks, after which they discussed what they had learned from the experiences. The students were then randomly assigned to verbalizing (V) or non-verbalizing (NV) treatment groups. They practiced the puzzle successively with two, three, four, and five disks. The V group reflected upon their experiences aloud to the researcher, whereas, the NV group reflected upon their experiences silently. The posttest was the six-disk puzzle. Efficiency scores were recorded for students of both groups. The V group proved to be superior in both the mean score and the number of correct subgoal moves.

Billings (January, 1987) used Logo as an instructional manipulative in conjunction with heuristics to develop problem-solving achievement and persistence in fifth- and sixth-grade students. The results of the study demonstrated that those students who learned problem solving and heuristics using Logo were better able to solve problems than those who did not use Logo. Previous research has supported this finding.

Logo and the microcomputer can be valuable aids in teaching problem solving to fifth- and sixth-grade students.

Testing and Evaluation of Problem Solving. If the horse should come before the cart, then surely the evaluation and testing of problem solving should be thoroughly researched before other variables related to problem solving are examined. Fortunately, there has been some progress in research related to testing and evaluation, but, perhaps, not nearly enough. The research of 1987 had looked at kinds of tests, kinds of problems in the instruments, and the relationship of these measures to other variables.

Barson (February, 1987) surveyed the types of questions being currently used in paper-and-pencil group tests of problem solving in selected states and school districts in the United States. He solicited the mathematics testing instruments from 50 state departments of education and 57 local school districts. A total of 130 tests used to assess problem solving in grades one through twelve was returned. An item analysis was used to ascertain if routine or nonroutine problems were used more frequently in the tests.

Routine exercises were classified as those that require direct recall or the application of a learned algorithm, including the use of an algorithm from the translation of a word problem. The nonroutine problems required more creative and critical thinking. A nonroutine problem can be one for which a learned algorithm might not be available. A pilot study was conducted in order to assure interrater reliability and to correct for experimenter bias in classifying the items.

The findings revealed that about 12 percent of the analyzed items required nonroutine problem-solving abilities, whereas, 88 percent tested students' abilities to solve routine exercises and perform routine calculations. It was concluded that the

evaluation instruments used by states and school districts were not testing nonroutine problem solving.

A national survey of instruments used to test for achievement in problem solving revealed that over 88 percent of the test items required only routine computations, whereas, about 12 percent required nonroutine problem solving behaviors.

Smith (October, 1987) conducted a longitudinal study to examine the relationships between achievements on five subtests of problem solving: Multistep, Multistep with Percent, Ratio/Proportion, Ratio/Proportion with Percent, and Percent, and three variables: Proportional Logic Structure, Structure of the Properties of a Group, and Cognitive Style Structure.

One hundred thirty-two students ranging in age from 12 to 15 were administered three tasks to assess their understanding of the structural variables. The three tasks were the Balance Beam Task to assess Proportional Logic Structure, a Figure Rotation Task to assess Structure of the Properties of a Group, and the Group Embedded Figures Task to assess Cognitive Style Structure. During the second year of the study, the students were reassessed on the first two tasks.

Regression analyses showed that (a) students who scored high on Proportional Logic Structure scored higher on each of the problem-solving subtests than those who scored low; (b) students who were rated as field independent on Cognitive Style Structure, i.e., were more analytic in their thinking, scored higher on the problem-solving subtests than those students who were classified as field dependent, i.e., were more global in their thinking; and (c) students who scored highly on the Structure of the Properties of a Group also scored highly on multistep and ratio/proportion subtests.

Hofmann (August, 1987) developed two forms of a paper-and-pencil, machine-scoreable, Four Step Problem Solving Test. The four steps of the tests consisted of (a) Read; (b) Select a Strategy; (c) Solve; and (d) Review and Extend. Five scores were determined, one for each step and a composite score.

Validity and reliability of the tests were studied with seventh- and eighth-grade students. Content validity was established with

a panel of experts, and reliability was determined using Kuder-Richardson Formula 20. Concurrent validity was determined by correlating the scores of the test with mathematics grades assigned by teachers.

Lewis and Mayer (December, 1987) explored the miscomprehensions of students doing arithmetic word problems, and Puchalska and Semadeni (November, 1987) studied the reactions of students to verbal arithmetic problems which have missing, surplus, or contradictory data. These are issues whose resolution would be of considerable interest to test makers in constructing instruments designed to evaluate problem-solving behaviors.

Lewis and Mayer felt that most of the troubles students experience in problem solving were due to comprehension difficulties rather than to problem calculational difficulties. In particular, they focused upon compare-type problems with consistent and inconsistent language. To illustrate the differences, consider the problem: (a) Joe has 3 marbles; (b) Tom has 5 more marbles than Joe; (c) How many marbles does Tom have? This is a consistent language problem. The unknown (e.g., the number of Tom's marbles) is the main subject of the second sentence, and the relational term in the second sentence (more than) is consistent with the required operation (addition). To illustrate inconsistent language, consider the problem: (a) Joe has 3 marbles; (b) He has 5 marbles less than Tom; (c) How many marbles does Tom have? In this case, the unknown is the object of the second sentence, and the relational term (less than) conflicts with the required arithmetic operation (addition).

Lewis and Mayer further hypothesized that problem solvers have a distinct preference for the order in which information is presented in a problem, with the preferred format corresponding to the order in consistent language problems. Furthermore, they felt that comprehension problems are more likely to occur when the problem format is not the preferred format.

They tested these ideas in two experiments with college students. In the first experiment, 96 students, ranging in age from 18 to 21, were studied, and in the second experiment, 26 college students similar to the first group were tested. The results showed quite conclusively that inconsistent language-compare problems are the source of major difficulties in comprehension. It was felt that students came to a problem solving situation with a set of preferences for the form of statements in compare-type problems. If this preferred arrangement was not present, the student rearranged the information which causes comprehension difficulties.

Puchalska and Semadeni (November, 1987) investigated the reactions of children to verbal arithmetic problems with missing,

surplus, or contradictory data (MSCD problems). The aim of this exploratory study was to expose children to MSCD problems during regular classes in grades 1 and 2. The experimental lessons were given in various schools around Warsaw, Poland, in May and June of 1985. The problems were presented orally to students with those in grade 2 also reading them. The children raised their hands and the teacher called on them successively.

Puchalska and Semadeni found that often students gave unsatisfactory answers to MSCD problems. Most often these children had not been taught to deal with this kind of problem; and, particularly, in a situation where pressures of peers may be quite pronounced. The authors felt that MSCD problems can be used to develop students' critical skills in reading problems. Furthermore, they felt that MSCD problems could be solved effectively only if several of them were presented to students consecutively. Isolated MSCD problems caused bewilderment.

Do you test your students with MSCD problems; that is, problems with missing, surplus, or contradictory data? If so, such problems should not be presented in isolation from other problems.

Concrete problems were considered by Caldwell and Goldin (May, 1987) and Carraher, Carraher, and Schliemann (March, 1987). Obviously, the kinds of problems used in a testing instrument will influence substantially the testing results.

Caldwell and Goldin compared the relative difficulties of concrete versus abstract and factual versus hypothetical problems with high school students in grades seven through twelve in a suburban Philadelphia school district. An abstract (A) problem describes only abstract or symbolic objects, a concrete (C) problem describes a real situation with real objects, a factual (F) problem merely describes a situation, and a hypothetical (H) problem not only describes a situation, but also describes a possible change in the problem that does not occur in the context of the problem (e.g., if there were six times as many boys then...).

The researchers found that in the junior high school classes the problems in increasing order of difficulty were CF (concrete factual), CH (concrete hypothetical), AH (abstract hypothetical), and AF (abstract factual). In the high school grades, the CF

problems were least difficult and the AH problems were most difficult. The CH and AF problems were of moderate but equal difficulty. The reversal of difficulty of AH and AF problems from junior high school to senior high school caused Caldwell and Goldin to conclude that teachers should not decide that abstract or hypothetical word problems are beyond the reach of students. Furthermore, they concluded, "It appears that the greater difficulty of hypothetical problems occurs almost exclusively in concrete situations" (p. 195).

Abstract or hypothetical word problems may be well within the reach of your students.

Drill, Practice, and Mental Computation

Drill and practice has been the subject of research efforts for many years (e.g., see Begle, 1979; Suydam and Dessart, 1980; and Dessart, 1983). During 1987, efforts on research in drill and practice were not extensive, probably because the interest in problem solving had tended to minimize the desirability of drill and practice as a learning tool. Research on mental computation has never occupied a central position in the research agenda of mathematics educators, although there was activity during 1987.

Drill and Practice. Campbell, Horn, and Leigh (Summer, 1987) compared the performance of two groups of third grade students in division of whole numbers. One group used a commercially prepared microcomputer drill program and a matched group used conventional, printed drill materials. The microcomputer program consisted of 12 diskettes designed to provide drill and practice for students of grades one through eight. The program would not permit a student to advance until mastery criteria had been attained. The printed drill materials consisted of mimeographed sheets of 50 problems from each level of the computer program. In order to avoid a possible Hawthorne effect, students of the print group were also given computer-based instruction in another area during the time of the experiment.

All students received 30 minutes of instruction each day. The computer group then received 20 minutes of practice and the print group was given worksheets for the same period. The study lasted five weeks during January-February 1984 in an affluent suburban school district of the Southeast.

The print group scored significantly higher on the first progress test; otherwise, there were no significant differences

between the two groups on any measures. The poorer performance of the computer group on the first progress test was probably due to the students' lack of familiarity with computers. The overall conclusion of the investigators was that there were no measurable differences between the two treatments. One might note here that much previous research indicates that computer-administered drill and practice is no more effective than teacher-administered drill and practice.

Campbell, Horn and Leigh noted that a disadvantage of the computer was the delay students experienced while waiting for the computer to finish reinforcement messages given by cartoon-like characters. Some students became restless and impatient. A disadvantage of the print-materials was the lack of immediate reinforcement. Students had to wait until a teacher corrected their papers for knowledge of results.

Faulk (January, 1987) compared four experimental groups: (a) computer drill and practice, (b) computer drill and practice plus counseling, (c) pencil-and-paper drill and practice with worksheets, and (d) paper-and-pencil drill and practice plus counseling, with students of the fourth and fifth grades over a six-week period. The pre- and posttests given to the students were achievement measures in computation. The paper-pencil treatments tended to increase scores more than the computer treatments. Some irregularities in the posttest were noted at one school, so these data were not used in the analysis of results.

Mental Computation. Mental computation was the topic of three investigations in 1987: Hope and Sherrill (March, 1987); Hope (November, 1987); and Scott (December, 1987).

The characteristics of unskilled and skilled mental calculators were investigated by Hope and Sherrill. They selected 15 skilled and 15 unskilled students from 286 students in grades 11 and 12 on the basis of their performances on a mental multiplication test. The test consisted of 20 mental multiplication tasks: ten easy (e.g., 30 x 200) and ten difficult (e.g., 32 x 64). Each item was presented on audio tape allowing 20 seconds between items, and the students could use pencil and paper only to record their answers.

Each of the 30 students performed 30 mental multiplication tasks (e.g., 25 x 48, 8 x 999, and 49 x 51), after which he or she was asked to explain the mental process of calculation for each example. These sessions were recorded on audio tape for later analysis. In addition, each student was given a Basic Fact Recall Test (BFR) which contained 100 multiplication facts from single-digit factors. The short-term memory capacity of

the students was tested with the forward and backward digit span subtests of the Wechsler Adult Intelligence Scale. A student's digit span is the number of digits in the longest sequence repeated without error.

Analysis of the mental multiplication tasks revealed four methods of solution and 12 calculational strategies. The unskilled students used strategies more closely related to written computation than mental computation, and the skilled students used strategies related to the numerical properties of the factors. The skilled students were able to recall more large products to aid in the calculation. Mental multiplication performances had a low correlation with measures of digit span memory.

Students skilled at mental multiplication of numbers use the numerical properties of the factors more often than students unskilled in mental multiplication.

Hope (November, 1987) studied the performances of Charlene, a 13-year-old highly skilled mental calculator of multiplication facts. Hope interviewed Charlene during which he gave her 50 multiplication facts which had been used in the Hope and Sherrill study. Each exercise was presented orally, after which Charlene mentally calculated the product and explained her methods, which were audio taped for later analysis.

Charlene used calculational methods which employed distributing, factoring, and direct recall of products. She had an extensive memory of squares of two-and three-digit numbers. Her digit span memory was much greater than average. She also had the ability to identify quickly prime and composite numbers. She claimed to be largely self-taught.

Hope advised teachers to encourage mental calculations with their students. He noted that pupils should not use paper and pencil for exercises such as, 57 - 8, 199 + 156, 1000 x 26, and 3535 ÷ 35. He felt that children should be encouraged to seek mental shortcuts and to use paper and pencil or a hand-calculator only when absolutely necessary.

Scott (December, 1987) developed a program which was based upon the Texas instructional program on number sense, mental computation, and mathematics. Ten minutes of each mathematics

period were devoted to this instruction with eighth-grade students. Mathematical application scores were significantly higher for these students when compared to those who had not received the training.

Attitudinal Factors

The attitudes of students toward mathematics and toward themselves in relation to mathematics are critical factors in learning mathematics. In fact, many teachers feel that attitudes are far more important than abilities. In any event, attitudes are an active area of research interest in mathematics education. In 1987, studies were conducted of attitudes of students toward mathematics, computers, tests, and other personal attitudes of learners.

Attitudes Toward Mathematics and Computers. Attitudes of students toward mathematics were investigated by Miller (April, 1987) and Alvi (March, 1987). Attitudes of students toward computers were studied by Gressard and Loyd (February, 1987) and Swadener and Hannafin (January, 1987).

Miller administered Aiken's Revised Math Attitude Scale (RMAS) to 329 seniors from a midwestern high school. The RMAS scores were used to partition the students into three categories, those who (a) liked mathematics, (b) disliked mathematics, and (c) were neutral toward mathematics. The students were also asked to identify in which of three grade intervals their attitudes toward mathematics developed: grades 1-6, grades 7-8-9, or grades 10-11-12.

Thirty students who claimed that their attitudes developed in grades 7-8-9 were interviewed by Miller. The 30 included five males and five females from each of the categories: "like," "dislike," or "neutral" toward mathematics. The interviews revealed that students' attitudes toward mathematics deteriorated as they moved to higher academic levels. The 30 students had studied a mean of 3.28 mathematics courses in grades 9-12. Their decision to study more mathematics than was required for graduation was related to their college aspirations. All 30 students had used calculators in high school and felt that the use of calculators did not influence their attitudes toward mathematics.

Alvi found no significant differences in attitudes toward mathematics of an experimental group of 68 students taught general mathematics in the ninth grade through an individualized instruction approach compared to a group of 21 students taught by conventional methods. The researcher found that the experimental students achieved significantly higher on the Comprehensive Test of Basic Skills than did the conventional group.

Gressard and Loyd measured the computer attitudes of three groups: (a) 161 junior and senior high school students who were enrolled in an accelerated language arts class; (b) 76 small liberal arts college students residing in dormitories; and (c) 119 community college students enrolled in a developmental mathematics course. The computer experiences of the students ranged from less than one week to more than one year.

Computer attitudes were measured by the Computer Attitude Scale designed by Loyd and Gressard. This scale produces subscales for computer anxiety, computer confidence, and computer liking. Math anxiety was measured by a modified version of the Mathematics Anxiety Scale designed by Fennema and Sherman.

For all three groups, computer experience accounted for a small but significant amount of the variance of the three computer attitudes: anxiety, confidence, and liking. The correlations between computer experience and computer attitudes were moderate and positive as were the correlations between computer attitudes and math anxiety. The authors suggested that math anxiety may be a minor but important factor in computer anxiety and computer confidence.

Swadener and Hannafin explored the attitudinal difference of 32 randomly selected sixth-grade boys and girls toward computers. The Computer Literacy and Awareness Assessment Instrument of the Minnesota Educational Computing Corporation (MECC), which is designed for secondary school students, was adapted in this study for sixth graders. The students were partitioned into groups according to gender and mathematics achievement, which had been measured by a mathematics skill test.

It was found that high-achieving mathematics students had expressed greater agreement with statements of computer confidence than the low-achieving students. Further, the investigators concluded that sixth-grade boys and girls responded with far greater similarities than differences.

Personal Attitudes. Band (February, 1987) explored the self-attributions for success or failure on a mathematical task of four groups of children: black and white females, and black and white males. Band reported that white children were more oriented toward success than black children. White males were more sensitive to failure than black males and white females, and black males were more stable with respect to failure than white males and black females.

Flower (January, 1987) interviewed 75 fifth- and sixth-grade students on the day that they received their graded mathematics test from their teachers. The children were asked questions related to their successes or failures on the test and the reasons for those test performances. The students had also been given a global measure of mathematics test satisfaction, prior to the mathematics test.

Efforts or lack of efforts were the most commonly cited causes of success or failure on the test. Further analyses revealed that students made primarily internal and unstable attributions.

Efforts or lack of efforts were cited most often by fifth and sixth graders as their reasons or explanations for good or bad test performances.

Perceived self-efficacy is one's judgments of how well one can deal with future situations. These judgments influence future courses of action and consequently should be highly relevant to academic achievements of children. Norwich (December, 1987) assessed theself-efficacy of 72 London, England, school children by asking them how certain they were of answering correctly a mathematics question and then rating their replies on an 11-point certainty rating scale. In addition, the researcher administered an inventory of mathematics self-concept to the children. Seven statements of the form, "I'm useless at math" or "I'm very good at math" were given in the inventory. The students also performed mathematical tasks (e.g., 5 x 57, 75 ÷ 5, 6^2, and 45 tenths = _____. The self-efficacy assessments were made on four occasions, with mathematical tasks performed by the children between the first and second self-efficacy assessments and between the third and fourth assessments.

Norwich reported that with low familiarity with the task, about 12 percent of the variance in mathematical performance was predicted and was due to mathematics self-concept and not self-efficacy. With greater familiarity, about 73 percent of variance could be predicted and again was due to self-concept rather than self-efficacy. This research pointed out the values of assessing self-efficacy, but it also suggested that better instruments to measure this construct are needed.

Juhl (March, 1987) compared the personal characteristics and identified the difference between 71 Vietnamese and 156 white

children who were achieving A's and B's in advanced high school mathematics courses in a suburban Washington, D.C. public high school.

Using discriminate analysis, Juhl found that the scale that best separated the two groups was "math as a male domain." Other scales that also separated the groups were: "math is just a set of rules" and "it is fun to study math." A scale that failed to separate the two was "parents really want me to do well in math." Juhl interpreted this last finding to imply that the scale was not unimportant but rather equally important to both groups.

SUMMARY

In this section, "Planning for Instruction," we have looked at research in six broad areas: historical development; aides and grades; teaching approaches; problem solving; drill, practice, and mental computation; and attitudinal factors. We have highlighted some "hints" or "ideas" from research. These hints or ideas do not necessarily enjoy greater validity than other findings, but represent key notions that may be useful or novel to the teacher, supervisor, teacher-aide, parent, or student.

The history of mathematics or the manner in which mathematics ideas have developed may influence the way in which these mathematical ideas are taught. Harper considered whether or not this phylogenetic development influenced the way children approached algebraic concepts. The Vietan approach was used by substantial numbers of students. DuPre and Nibbelink, Stockdale, Hoover, and Mangru looked again at the modern mathematics movement. DuPre concluded that UICSM had been criticized unfairly, whereas, Nibbelink and his colleagues noted that problem solving declined in grades four, five, and six during 1965-1975. They felt this was due to the emphasis upon precision of language and deduction instead of problem solving during the new mathematics period.

The use of teacher aides for middle school general mathematics classes is highly desirable and important according, to research by Palmer. MacIver wondered why so many upper elementary school pupils felt that they were mathematically competent and how reliant they were upon adult assessments for these positive self evaluations. He found that frequent grades by teachers highly influenced the reliance of students upon adult assessments.

Zepp discussed the Whorf Hypothesis which contends that a student's thoughts and logical processes are dependent upon the student's first language. Zepp's research did not provide substantial

support for this hypothesis. Bradley compared student achievements in traditional and transformational geometry. Surprisingly, the transformational students showed greater gains in their understanding of formal geometry.

There were many studies related to problem solving in 1987. Researchers studied the many heuristics and strategies of problem solving, and, in general, found that emphasizing heuristics and strategies helped students become better problem solvers. The testing of problem solving is in need of serious efforts by researchers. Barson found that 88 percent of the items on tests designed to tap problem-solving abilities required only routine computations. We cannot make significant progress in problem-solving research and in classrooms until we develop instruments that will accurately measure this important ability.

Research on drill and practice, which was the focus of much research in the 1920s, was of less interest in 1987. The research interest had turned more toward the use of computers to aid in drill and practice. A study by Campbell, Horn, and Leigh did not provide evidence to support the contention that the computer is an important asset to effective drill and practice with third-grade students. Children who are adept at mental calculations of products appeared to rely heavily upon the numerical properties of the factors, according to research reported by Hope.

Attitudes toward mathematics, computers, and self-attitudes were investigated in 1987. It appeared that computer attitude is closely related to mathematics anxiety. Consequently, if we wish to improve attitudes toward computers, we should work on improving attitudes toward mathematics. Self-efficacy or a student's judgment as to how well he or she can perform in a future mathematical situation would appear to be an important construct in improving the learning of mathematics. Research by Norwich did not appear to support the value of this construct in predicting mathematical performance. Better measures of self-efficacy are needed to insure progress in research on this crucial notion.

II. MATHEMATICAL CONTENT AND MATERIALS

In this section on mathematical content and materials, we shall review research related to three broad categories: (a) the structure and organization of content, (b) methods of instruction related to content, and (c) instructional materials in mathematics.

The structure and organization of content must be related to the overall objectives of a school program, if instruction in mathematics is to fulfill its proper role in the total instructional program of a school. Hughes studied a small elementary school to determine if its instructional program was meeting the objectives formulated by the district and found some surprising results. The structure of content must also serve special students.

Research on methods of content instruction included attention to counting, early number concepts, measurement, geometry, algebra, functions, and probability. Research on counting is popular. The proper interpretation of the equals sign by children is still being debated by educators. The concept of variable and its development continued to be a topic of primary focus. Function interested researchers for many, many years, and this interest continued during 1987. Many teachers feel that probability needs greater emphasis in schools. Probability is often difficult for students. The complexities of learning probability should be studied by mathematics educators.

Finally instructional materials provide the delivery system for student learning. Textbooks, supplementary teaching materials, calculators, and computers were studied for their effectiveness in promoting learning. Computers, in particular, received a great deal of research attention.

Structure and Organization of Content

The structure and organization of mathematical content are crucial to success in teaching that content. During 1987, studies explored the structure and organization of content in elementary school settings, the structure of content for special students, both the able and the less able, and finally the effects of language and such notions as "variable" as they relate to structural questions.

Elementary School Content Structure and Organization. Hughes (October, 1987) studied a typical small Midwestern elementary school to determine if its mathematics program was meeting the established mathematical objectives of the school district; Wheeler (August, 1987) traced the genesis, development, and implementation

of the Comprehensive School Mathematics Program (CSMP) for elementary schools; Katz (July, 1987) investigated understandings of early number concepts of elementary children identified as at-risk and not at-risk for future academic achievement; and Avesar and Dickerson (October, 1987) investigated the one-to-one correspondence structure in four- and five-year-old children.

Hughes selected a small Midwestern elementary school of kindergarten through grade six enrolling 506 students to determine whether or not the school's program was satisfying the cognitive mathematical objectives of the school district. She analyzed the results of the Metropolitan Achievement Test (MAT), administered to the children in April, and the criterion-referenced tests of the U-SAIL Mathematics Program. She concluded that the MAT did not measure enough objectives to adequately assess achievement of the district's mathematics objectives. The U-SAIL test results indicated that objectives were adequately met in kindergarten, first, and second grades but not in the third, fourth, fifth, and sixth grades.

A study of a typical, small, Midwestern elementary school of grades K through 6 revealed that mathematical objectives were being adequately met in grades K, one, and two but not in the remaining grades.

The Comprehensive School Mathematics Program (CSMP), which was popular during the period of 1966-84, attempted to organize the mathematical content of the elementary schools so that it would reflect a spiral sequence and would introduce applications appropriate to the child's level of understandings and interests. Non-verbal languages, e.g., Language of Arrows, Mini-Computers, and Strings, were included to teach concepts and problem-solving methods. Wheeler's study of CSMP revealed that when it lost national funding, it gradually disappeared from school programs. One might note that this has often been the case with novel programs. Their lives seem so highly dependent upon special fundings which lead to their demise when the fundings are withdrawn. Even so, perhaps, such novel programs do have a telling effect upon improving traditional programs.

Katz investigated informal number understandings of children identified as at-risk and not at-risk for further academic achievement.

The children in the study were 186 kindergartners enrolled in three Long Island schools. The children were classified by their performances on the Hainsworth and Hainsworth Preschool Screening System into four potential groups: (a) high achievers; (b) average achievers; (c) at-risk achievers; and (d) discrepant achievers, that is, children who performed unevenly in the assessment. Five children were randomly selected from each of the four groups and were evaluated on four tasks: (a) counting, (b) application strategies, (c) concepts, and (d) underlying cognitive structures, e.g., conservation and class inclusion. Katz found that at-risk children and some discrepant children demonstrated less developed informal skills, knowledge, and strategies than the not-at-risk children. These differences were especially pronounced on counting and application task assessments.

Many will agree that an understanding of one-to-one correspondence is crucial in the development of early number concepts. Children's understanding of one-to-one correspondence should influence the sequencing of mathematical content for children. Avesar and Dickerson conducted three experiments with four- and five-year-olds. The methods employed by children were assessed by their responses over a series of problems in which sets were arrayed in parallel rows. When lines connected the elements in rows, nearly all children used one-to-one methods spontaneously; however, when pairs of elements were enclosed in the windows of a grid, one-to-one plans were only used after the children received experiences in pairing the elements. The researchers concluded that most four-year-old children have one-to-one plans in long-term memory but fail to use these plans in some perceptual support systems, e.g., windows instead of lines connecting pairs of elements.

Structure of Content for Special Students. Snider (January, 1987) studied gifted programs in mathematics for junior high/middle school students. He concluded that the major key to a successful program is an outstanding teacher who provides differentiated instruction for several levels of the gifted. Furthermore, the program should contain both accelerative as well as enrichment options. Thinking and problem-solving skills should be emphasized, and opportunities for in-depth study should be provided. Chances for academic competitions should also be made available to gifted students.

Kelly (March, 1987) compared the effectiveness of two curricula for teaching basic fraction skills to mildly handicapped and remedial high school students. Twenty-eight students were matched on mathematics achievement and randomly assigned to one of two treatments. One treatment included explicit strategy teaching and discrimination practice using an interactive video disk presentation. The second

treatment used the basal textbook. Teachers in both treatments emphasized clear demonstrations, guided practice, and special classroom management procedures.

The video disk curriculum resulted in significantly greater posttest and maintenance test scores than the basal textbook curriculum. Levels of on-task behaviors were higher in the video disk treatment, but there were no significant differences in the two treatments on independent worksheets completed by students.

A perennial question for high schools is the debate as to which should come first, Geometry or Algebra II. Nichols (February, 1987) randomly partitioned students into two groups, one which studied Geometry followed by Algebra II and the second which studied Algebra II followed by Geometry. To compare the results of both sequences, measures of achievement (both standardized and teacher-constructed tests), attitudes toward mathematics, and growth in thinking skills were made. Continuation of students in mathematics was determined by comparing the percentages of each group who enrolled in senior mathematics courses. The results indicated no significant differences between the two sequences, but the findings did indicate that the subject taken "last" had the higher achievement growth.

The often asked question of which is better, Geometry followed by Algebra II or Algebra II followed by Geometry, is still unanswered by research!

Verbal Behavior and the Concept of Variable. The effects of language in learning mathematics were studied by Miura (March, 1987) and Smith (January, 1987). Investigations related to the concept of variable were conducted by Crook (January, 1987) and Comstock (January, 1987).

There is little question that Asian Americans have demonstrated superior performances in mathematics. Miura conjectured that national language characteristics may be partly responsible for the mathematical superiority of the Asian Americans. Miura noted that Asian languages having their roots in Ancient Chinese (e.g., Chinese, Japanese, and Korean) are organized so that names of numbers are congruent to numbers of the Base Ten Numeration System. In the Asian languages, "eleven" is spoken as "ten-one," 17 as "ten-seven," and 80 as "eight-tens," which is quite different

than spoken number names in French, German, and English. Miura suggested that the Asian language names for numbers might assist children in developing stronger cognitive, number structures than number names in other languages.

Miura compared a sample of 21 students, nine girls and 12 boys, who attended Japanese Saturday school in San Franciso, were fluent in Japanese, and came from homes where Japanese was spoken. This sample was compared with 20 students, 10 girls and 10 boys, who attended a private grammar school in San Francisco and did not speak Japanese. Miura's findings suggested that there were differences in the two groups of children. The Japanese-speaking children used canonical base ten constructions (i.e., a construction that used the correct number of ten blocks and unit blocks with no more than nine units in the units position) more often than the English-speaking children. This, in turn, may have provided the Japanese-speaking children with a numerical advantage in learning mathematics.

The Chinese, Japanese, and Korean languages use number names that are congruent with Base Ten representations, e.g., "eleven" is spoken as "ten-one." This fact may provide a cognitive, mathematical advantage for Japanese-speaking youngsters!

The language used by teachers might seem to affect student achievement and attitude. Smith (January, 1987) investigated the effects of language uncertainty (e.g., maybe, might, sometimes, various, sort of, almost, about), language bluffing (e.g., actually, frankly, so to speak, you know, in essence, in a nutshell, obviously), and student participation on achievement and attitudes. One might conclude that vague terms included in uncertain and bluffing language might lead to mental confusion for students, thus resulting in lower achievement in and attitudes toward mathematics.

A sample of 96 sixth graders with approximately 90 percent Caucasians and 47 percent females was selected. Each student was randomly assigned to one of eight groups which were defined by the eight possible combinations of two uncertainty conditions (uncertainty, no uncertainty), two bluffing conditions (bluffing, no bluffing), and two participation conditions (participation, no participation). Each group was given a 20-minute audiotaped lesson while they observed overhead projections and demonstrations of

the content which consisted of concepts from elementary topology concerning transversibility of curves (e.g.,, Euler's formula for networks in a plane). The frequency of uncertainty and bluffing terms were controlled at levels representing typical teacher usage. The students were tested immediately following the 20-minute lessons. Student participation involved either completing or not completing handouts during the lesson.

Smith reported that overall student achievement was not significantly affected by uncertainty, bluffing, or participation, but significant differences were found in student perceptions and attitudes. In general, uncertainty and bluffing terms caused decreases in student achievement and attitude on certain test items. Similarly, decreases were also associated with non-participation of students. Nevertheless, language usage by teachers is a factor deserving greater research attention.

Crook studied the effects of BASIC programming on seventh-grade students' abilities to use and understand the concept of variable. Two student groups were formed randomly, one receiving 15 one-hour sessions in BASIC and the other receiving 15 hours of classroom instruction. The treatments were reversed after a midtest. Crook concluded that some concepts of variable were learned by programming, and that the programming also enhanced knowledge of variable that students had gained through classroom instruction. Neither sequence, programming followed by classroom instruction or classroom instruction followed by programming, proved superior.

Comstock investigated the development of the concept of variable among seventh graders during their introduction to the use of letters to represent numbers. Comstock found that these students initially thought of a letter as representing a specific but unknown number. Later students were able to describe a letter as representing any number, but then were unable to use a letter as a symbol for operations. The research seemed to support the conclusion that so often success in learning mathematics is a matter of the cognitive level of development of students.

Methods of Content Instruction

Research during 1987 dealt with childrens' experiences in counting; early number relations and operations; measurement and geometry; and instruction related to algebra, functions, and probability. Each piece of research had implications for the improvement of instruction in mathematics at various levels.

Counting. Historically as well as psychologically human experiences with number began with counting. In attempts to

better understand how children learn arithmetic and mathematics, it seems highly appropriate that counting should be the focus of research attention.

Baroody (March, 1987) conducted structured clinical interviews with 10 girls and 7 boys ranging in age from 4 years, 11 months to 6 years, 7 months, with a median age of 5 years, 4 months. During the nine-month study period, Baroody focused upon (a) the learning of a concrete counting (CC) strategy for addition, (b) the transition from concrete to mental strategies, and (c) the role of commutativity in counting.

Baroody was surprised to find that all but three of the 17 children had to be shown a CC strategy when presented with an addition task, e.g., "5 + 1" typed horizontally on a card. However, all but five children learned a CC strategy quickly. The remaining five required as many as 12 to 21 demonstrations to learn a strategy.

Baroody found three types of mental counting strategies in children following their mastery of a CC strategy: (a) counting all, starting with the first addend, (b) counting all, starting with the larger addend, and (c) counting on from the larger addend. A knowledge of commutativity of addition didn't appear to be necessary for the children to employ more economical addition strategies depending upon commutativity.

Kindergarten children had to be shown counting strategies when presented with a simple addition task, but most learned such strategies quite quickly.

Mitchell (March, 1987) varied the typical counting situation for children by presenting them with items involving the concept of owing something to another person (e.g., You have been promised five candies. Here are some candies (three are given to the child). How many candies do I owe you?). A sample of 28 day-care children, ranging in age from three years, eight months to six years, three months were presented "owing" problems.

The children were given a problem in which the number of candies was handed to them, and they were requested to determine how many candies were still owed them. Problems were presented in which one, two, or three candies were owed. The number of

correct responses tended to increase as the owed numbers became smaller. If the child's response was incorrect, he or she was given a tray of seven candies to aid in solving the problem. The successful children either took one candy at a time from the tray until the correct sum was obtained, or paused to recount the candies in their hands. In any event, counting methods were employed by the children to solve the owing problems.

Adey (October, 1987) explored the possibility that task demands may mask the conceptual awareness of the necessity to employ counting methods when addition situations are presented to young children. This conjecture may explain why Baroody found that many children had to be shown concrete counting procedures in the presence of addition situations. Adey's sample consisted of 40 children, 20 three-year-olds and 20 four-year-olds, from preschool and day-care centers in Adelaide, Australia. Adey found that when children received both visual and tactile feedback during addition problem situations, their counting performances greatly improved over occasions when they did not receive feedback. Furthermore, Adey observed that preschool children understood that items can be counted in any order before they appreciated or intellectualized that a sum is not changed by the order of counting of items.

Preschoolers' apparent inabilities to use counting methods in response to addition situations may be masked by the complexities of the task demand.

Yarmish (February, 1987) studied the abilities of five- to ten-year-old Down's Syndrome children to perform certain equivalence tasks in the presence of certain potentially confounding task variables. Yarmish found that perceptions of shape, color, and area interfere with the child's perception of number. A positive correlation was found between performance and age. Results suggested that Down's Syndrome children may not be as limited in perceptions of number as had been previously assumed.

Newman, Friedman, and Gockley (October, 1987) studied 38 children (19 kindergarteners and 19 first graders, 20 boys and 18 girls) with a mean age of 6 years, 2 months. All children could count accurately by ones to at least 36. The children were presented stimulus displays of 12, 24, or 36 dots in a random pattern , clustered format, or rectangular array on an Apple II

microcomputer monitor. The children were shown 27 displays: nine consisted of random arrangement of dots; nine consisted of clusters of 2, 3, or 4 dots each; and nine consisted of rectangular arrays of 2, 3, or 4 dots per column. Each child was asked how many dots appeared on the screen, after which their responses were recorded.

The researchers found that children were quite accurate in enumerating small-numerosity and non-random displays. They were likely to report counting by multiples, rather than ones, on small-numerosity and clustered displays. The children also counted by multiples of twos, threes, or fours that corresponded to the number of dots in subgroupings. It appeared that clustered arrangements stimulated more multiple counting than rectangular arrangements, especially when the total number of objects was large and the size of subgroupings were small. There was a tendency to count by ones large-number displays which were arrayed randomly or in rectangles; however, the children were more likely to count in multiples when given large displays that were clustered.

Kindergarteners and first graders have a tendency to count large displays of dots by ones when the dots are arrayed randomly or in rectangles, but count by multiples those large displays that are clustered in smaller subgroupings.

Cowan (June, 1987) conducted experiments of counting with children of a London, England nursery school. He concentrated upon presenting the children with cards on which dots were arrayed in rows and in various colors. The children were instructed to count the number of dots on the cards. The children were classified by levels depending upon their counting success.

Cowan reported a variety of interesting findings. In the first experiment he found that children judged (a) equal-number, unequal-row-length displays more accurately than (b) unequal-number, unequal-row-length displays, which in turn were judged more accurately than (c) unequal-number and unequal-row-length displays. In the second experiment, (c) was judged least successfully, as in the first experiment, but (a) and (b) were judged with about the same level of successfulness.

Early Number Relations and Operations. In 1987, research was done related to children's interpretations of the equals

sign, their judgments related to commutativity, their experiences with addition and subtraction strategies, and their understanding related to fractions, decimals, and rational numbers.

Most children probably interpret the equals sign in a number sentence as an instruction "to do something," e.g., 2 + 3 = _____ implies the child should add 2 and 3 and write "5." One would hope that children would advance to the stage of interpreting the equals sign as "the same as," e.g., 2 + 3 = 5, means "2 + 3" is the same as "5." Cobb (May, 1987) studied 34 children from five classrooms during May of their first school year. He felt that there would be differences in equals sign interpretations of children who were subjected to more codified, textbook learning experiences than to self-generated methods of arithmetical learning, particularly in relation to three recent models of early number development. Cobb also interviewed the teachers of these children to determine their reliance upon the textbook, their teaching methods, and their sensitivity to self-generated learning methods of students.

Cobb concluded that the "do something" interpretation of the equals sign appeared to be deeply rooted in social interactions with the teacher, that is, when confronted with "7 + 3 = _____," the student does something that the teacher wants completed, that is, produces the answer, 10. He felt that there is little reason to stress the "same as" meaning of equality until children can construct appropriate relations between numbers as objects, express numerical understandings, and develop conceptual number structures. He also noted that "If there is one lesson to be learned from this analysis, it is simply the value of teaching by negotiation" (p. 122).

In order that children might experience more self-generated methods of learning arithmetical operations, teachers might employ more "teaching by negotiation," i.e., a concerted effort to mesh the goals of pupils and teachers.

Comiti and Bessot (February, 1987) described an activity involving French children in classes equivalent to the second grade in the United States. The teacher chose two numbers, which were then written on cards. The numbered cards were given to two children, who attempted to determine which student had the larger number by writing questions on a sheet. No talking was

permitted, and the only forbidden question was, "What is your number?" The research hoped to use an epistemological approach to the study of numeration, to create a situation which would encourage students to use numeration rules for solving a problem, and to eliminate the students' dependence upon outside assistance.

Comiti and Bessot reported two major strategies: (a) the student attempted to find the unknown number (e.g., "Is 15 your number?") and (b) the students attempted to order the two numbers (e.g., "Is your number larger than 35?"). It was felt the children's knowledge of numeration underwent change during the activity. The children not only established links between numeration rules and number orders, but they made these links operational.

Baroody (January, 1987) evaluated commutativity as understood by 34 moderately retarded students ranging in age from about six to nearly 21 years. He grouped the students into four groups according to training in addition: (a) no training; (b) minimal training; (c) moderate training; and (d) regular training. The students were interviewed individually outside their classrooms.

The interviews essentially involved having the student observe that 5 + 2 = 7, e.g., five cookies and two cookies make seven cookies, and then asked whether or not 2 + 5 would also make 7. A total of 16 trials were conducted over two days in which both commuted and noncommuted addition pairs of numbers were presented.

About 50 percent of the retarded students who had or were receiving computational training understood commutativity as a general principle. The results also suggested that even moderately retarded students could learn or discover this principle.

Beentjes and Jonker (Fall, 1987) investigated inconsistencies in addition and subtraction strategies with 168 second and third graders in six Dutch elementary schools. For this study, consistency was defined as solving comparable sums using the same strategy. The researchers considered three questions: (a) How many children were inconsistent and why? (b) Were inconsistencies tied to specific tasks? and (c) Did inconsistencies lead to irregularly appearing errors?

In response to the first question, Beentjes and Jonker found that almost 50 percent of the children showed inconsistencies. The inconsistencies were not concentrated in certain types of exercises but were found in nearly every set. In regard to the second question, it was found that inconsistencies were more prevalent in unfamiliar types than in familiar-type exercises. Thirdly, inconsistencies were associated with certain strategies, for example:

42 - 15 = __		46 - 23 = __
40 - 10 = 30		40 - 20 = 20
5 - 2 = 3	or	6 - 3 = 3
30 + 3 = 33		20 - 3 = 17
so, 42 - 15 = 33		so, 46 - 23 = 17.

Since children's uses of strategies varied with the same exercise, inconsistencies occurred irregularly.

Carnine, Engelmann, Hofmeister, and Kelly (Winter, 1987) described a videodisc program entitled Mastering Fractions, which consisted of six steps designed to assist teachers in diagnosing and remedying student problems. The steps were:

(a) The narrator posed questions, to which the students responded orally. The classroom teacher monitored and assisted students.

(b) Immediately following the demonstration, the students worked on exercises which the teacher monitored. The last exercise was an informal test which was followed by a remedial disc lesson if more than 20 percent of the students failed the exercise.

(c) Students did homework without teacher help.

(d) The next lesson began with a short quiz, followed by specific videodisc remediation, if needed.

(e) Every fifth lesson was a quiz, which was again followed by specific remediation as may have been needed.

(f) The learned skill was reviewed on every or every other lesson or used later in a more complex skill.

This highly defined program was compared with a basal textbook approach which began each lesson with an introduction followed by pages of explanation and independent work by students. The videodisc program was designed to insure student attention which may or may not have been attained with the basal textbook.

Ten lessons from the interactive videodisc course, Mastering Fractions, were compared with 10 comparable lessons from Mathematics Today, published by Harcourt Brace Jovanovich. The students were 32 mildly handicapped and remedial high school students randomly assigned to the two methods. The students were pretested, posttested, and maintenance-tested. The results of the testing appeared to

favor the videodisc treatment, and the researchers felt that its superiority was due to the instructional design features of the videodisc program.

Interactive videodisc programs were successful in teaching concepts of fractions to mildly handicapped and remedial high school students.

Greer (January, 1987) reported the results of a test covering calculation, estimation, and conceptual understanding of decimals given to 65 12- and 13-year-olds in a school near Belfast, Northern Ireland. The results revealed that: (a) many of the children did not understand the ordinal properties of decimals (e.g., only 45 percent identified 0.62 as the largest of the three numbers: 0.62, 0.236, 0.4); (b) the misconceptions that multiplication always produces larger, division always produces smaller, and the larger number should always be divided by the smaller were very prevalent; and (c) merely changing the numbers in a multiplication or division problem changed dramatically the choice of operation by the students.

Greer also investigated nonconservation by interviewing three students at a time while they worked a set of problems involving decimal notation. The children had been identified previously as being possible nonconservers. It was found that conservation for decimals seemed as difficult to induce as most standard Piagetian conservation tasks.

D. A. Smith (April, 1987) investigated rational number understandings in third-, fifth-, and seventh-grade students. Various embodiments of fractions were tested: word, set, and region. Performance on region embodiments tended to be lower than other embodiments. Across embodiments the most difficult judgments involved numerically different but equivalent fractions (e.g., 1/2 and 2/4). Many of the judgments involved different sized regions. The overall results suggested that poor performances in the region embodiments were due to inaccurate or incomplete problem representations.

Measurement and Geometry. In this section, we will review research related to measurement, the learning of geometry using Logo, the development of the concept of angle in children, and van Hiele levels and geometric thought.

Boulton-Lewis (November, 1987) attempted to determine children's sequential knowledge of measuring of length. Three different sequences of length measuring knowledge were defined. The first was a logico/mathematical sequence of skills based upon measurement theory. Skills were determined and ordered according to their being prerequisites for subsequent skills. The second sequence of skills was ordered according to the ages at which 50 percent or more of children could master the skills, as determined by a review of the research literature. The third sequence was based upon analyses of children's capacities to process information related to skills. The three sequences were designated as Logico/ Mathematical, Literature, and Information Processing Sequences.

Boulton-Lewis tested 80 children from kindergartens and primary schools of Adelaide, South Australia. The tests included a memory space measure in which children were asked to recall the numbers of colored squares on cards presented to them in a particular order. The children's short-term memory was measured by determining their abilities to repeat numbers and words presented to them. The children were further tested on a series of some 22 variables ranging from their abilities to quantify the numerosity of small sets without overt counting to constructing horizontal lines and measuring them with a 30-cm ruler.

The analyses of testing of the children revealed that the sequence for acquisition of length knowledge was closely related to the children's abilities to process information. Increasingly complex knowledge of measuring was associated with increasing M-space and short-term memories. Such knowledge as invariance of length and cardinal number appeared to be closely related to information processing capacities of children. The researcher suggested that the curricular sequence for length measuring should emphasize the logico/mathematical sequencing with special attention to M-space, short-term memory, and capacity to process information.

Logo has become popular in many elementary schools. Noss (November, 1987) explored children's conceptions of length and angles with 84 children who had learned Logo and 92 who had not. The children were between eight and 11 years old. In Logo, the ideas of length and angle are related directly to the commands of FORWARD and RIGHT.

The children were distributed among five classrooms in five schools. Each class was equipped with one computer, a printer, and a floor turtle. The teachers taught their classes and accommodated Logo with the on-going class activities. The Logo work was not linked to their mathematical studies, and geometry was not explicitly taught to the children. The students worked about 75 minutes per week on projects related to Logo.

Children who had used Logo and those who had not were tested after a year with a 12-item test, six items related to the concept of length and six related to angle. The items were designed to tap length conservation, combination (e.g., additive property of length), and measurement; right angle conservation; and angle conservation and measurement.

The results showed a positive effect of Logo upon children's understandings of length conservation and the concept of unit of measurement. The results for angle understandings were far more pronounced, with significant effects in favor of Logo on the students' conceptions of angle conservation and measurement. The Logo students were significantly better than non-Logo children at ignoring the orientation of angles and side length when judging the relative measures of the sizes of two angles.

Research suggests that Logo experiences may enhance children's understandings of angle conservation and angle measurement.

Robinson (January, 1987) focused upon the effects of length of the sides of an angle and/or the rotation of the angle upon students' perceptions of angle measurement. Fourteen third and fourth graders participated in the study. Three methods were used to assess the children's understanding of angle: visual perception, manipulative construction, and Logo programming. Scores were obtained for the children, and levels of development were established based upon the score totals for each child.

It was found that these levels of development correlated significantly with achievements on a standardized mathematics test but not with age of the children. It was also felt that the levels of development would be useful in devising a curricular sequence for angle development.

The theory of van Hiele levels of geometric thought was the subject of two investigations: Kay (February, 1987) and Denis (October, 1987). Kay concluded that the van Hiele theory may not describe adequately the full complexity of the ways in which young children understand geometric thought. In particular, she felt that the van Hiele theory may describe the development of concepts when instruction proceeded from "specific" to "general," but not when instruction went from "general" to "specific." She

also concluded that regardless of instruction, some geometrical concepts will be initially understood best through induction and others best through deduction. Denis compared the relationships between Piagetian stages of cognitive development and van Hiele levels of geometric thought among 156 Puerto Rican adolescents who had studied high school Euclidean geometry. The researcher concluded that the geometry curriculum should be revised to reflect more closely the van Hiele levels, and provisions should be made to ascertain the Piagetian levels of development in students, although the relationships between the two theories did not appear stable.

Algebra, Functions, and Probability. Research related to the teaching of algebra, function concepts, graphing, and subjective probability will be discussed in this section. All three of these are crucial topics in the mathematical development of students and are most worthy of the attention of researchers. Al-Ghamdi (August, 1987) explored the uses of microcomputers in learning algebraic concepts, Petkovich (June, 1987) experimented with teaching algebra with worked examples, and Norman (September, 1987) studied unitizing strategies in algebraic and graphical contexts.

"My Dear Aunt Sally" (multiply, divide, add, and subtract) is a mnemonic convention that has been used by many teachers for teaching the order of operations. Al-Ghamdi found that the time spent teaching the topic of order of operations including conventions may be inadequate. Al-Ghamdi also compared the teaching of order of operations using microcomputers with traditional methods with 132 students of three Florida high schools. It was found that the microcomputer approach was more effective than traditional methods as measured by tests of achievement, retention, and transfer.

Petkovich had 117 ninth-grade algebra students, who had failed a pretest on equation solving, study one of four sets of worked examples on algebraic equation problem solving. The students also studied memory retrieval cues. They were tested immediately, 15 days, and 30 days after instruction. The students who studied a wide range of examples were capable of generalizing what had been learned to more complex problems.

Norman studied the unitizing strategies of 43 ninth-grade algebra students, 26 prospective elementary and middle school teachers, and 9 preservice secondary mathematics teachers. It was found that some students did not unitize in certain algebraic situations, but when students did unitize, they used one of there methods: instrumental, recognitive, or operational.

Function was studied carefully by Davidson (December, 1987). He defined three categories of tasks to represent functions: (a)

functions defined as exchanges of properties (e.g., completing a path through tunnels which involved exchanges of routes); (b) functions defined as rotations of regular polygons (e.g., predicting the location of the vertex of a colored square after rotation according to pictures of various configurations); (c) and morphisms (e.g., matching wooden triangles to colors, while preserving cyclical relations among dots). In addition, numerical tasks and logical tasks were also devised.

Seventy-two children aged five years to nearly eight years of age were tested and videotaped in a laboratory setting. Three different sets of materials were used in each testing to study functional, numerical, and logical concepts. Simpler problems were used for lower ability children and more complex for higher ability children.

Davidson found a difference between five- and seven-year-old children. Trial-and-error procedures in five-year-olds were replaced by anticipatory or inferential strategies in seven-year-olds. In the tunnel task, this transition happened at about 6.5 years of age. It was found that numerical and logical performances were related to functional ability, whereas, displacement functions were associated with logical but not numerical abilities.

Mathematical graphs were investigated by Vonder Embse (November, 1987) and Curcio (November, 1987). Vonder Embse compared expert and novice graph readers. Fifty students viewed five graphs of polynomial functions on a computer monitor. The viewing area was partitioned into 99 blocks of equal size. The blocks were identified as important or less important depending upon what they contained, e.g., a block containing a maximum value was considered important. Two variables were measured: (a) average length of fixation for each block, and (b) percent of total time spent in each block.

It was found that expert graph readers had significantly greater average fixation in important blocks than novices; otherwise, differences between novices and experts were not significant. Vonder Embse recommended that students should be provided with substantially greater experience in reading, interpreting, and using graphical information.

Curcio tested 204 fourth graders in New York City schools for their comprehension of graphs, prior knowledge of mathematics and graphical forms, reading achievement, and mathematics achievement. Reading achievement, mathematics achievement, and prior knowledge of graphical forms were the best predictors of graphical comprehension for the fourth-grade students. Curcio also recommended more graphical experiences for children.

Researchers recommend that students be given more experiences in reading, interpreting, and using information presented in graphical form.

Huber and Huber (December, 1987) determined whether or not subjects would apply six formal principles of mathematical probability when confronted with two kinds of problems: a gambling task with objective probabilities and a sporting task without objective probabilities. The six formal principles varied from transitivity (i.e., If event A is more probable than B and B is more probable than C, then A is more probable that C) to complements (i.e., If event A is more probable than B, then the complement of B is more probable than the complement of A).

They experimented with 144 Austrian subjects ranging in age from nearly four years to 19 years. The subjects were partitioned into 14 groups according to ages and presented individually with the two tasks. Their reactions to these tasks were used to infer their intuitive understandings of the formal probability principles. Huber and Huber found that the subjects were able to apply the formal principles, without verbalizing them, and that these principles appeared to be a better framework for subjective probabilities than numerical probabilities. This latter finding may not agree with the current and somewhat popular belief that probability should be initially introduced with a numerical study of data sets.

Mathematical Instructional Materials

Textbooks, supplementary materials including calculators, and computer uses were studied by researchers during 1987. The textbook is very often the core of the instructional program in many schools. In addition, large sums of money are spent each year by school districts in purchasing textbooks. In spite of these facts, the number of studies related to textbooks has never been large. Supplementary materials and especially calculators are being used in schools but perhaps not as extensively as they should be. Research interest in computers was very significant in 1987, with a large number of studies. We shall consider research related to textbooks, supplementary materials, and computers.

Textbooks. Researchers analyzed textbooks to determine how much of the content was new, compared and analyzed the coverages

of textbooks, made comparative evaluations, and explored ways that books might be enhanced.

Flanders (September, 1987) analyzed three popular textbook series for K-8 and the algebra books published by the same companies to determine how much of the content of those books would be new material for students. Each page of the textbook was classified as new, old, or neutral. New material appeared in lessons, in exercises, or in enrichment materials. If any page of the textbook contained any new material, even a single exercise, it was classified as new. Pages containing only pictures, chapter reviews, and chapter tests were classified as neutral. Supplementary materials at the beginning or end of the textbooks were not included in the analyses.

Flanders found that students of grades two to five encounter about 40 to 65 percent new material in the textbooks examined. He felt that this was equivalent to two or three days of instruction each week. By the eighth grade, the amount of new material had dropped to 30 percent or about 1.5 days of instruction each week. On the other hand, about 88 percent of the material in algebra books was new.

An analysis of three popular textbook series revealed that only about 30 percent of materials in eighth-grade textbooks was new, whereas, 88 percent of the material in algebra books was new.

Mehrens and Phillips (Winter, 1987) assessed the curricular validity of the Stanford Achievement Test by quantifying the overlap of the test items and the content of the textbooks used in grades five and six of a Midwestern school district. A 180-cell matrix classification method was used to analyze the test and textbooks. The content in the textbooks covered almost all of the 53 cells in the matrix assessed by the Stanford Test.

Black (March, 1987) analyzed five of the most popular elementary mathematics textbooks to determine the teacher and student behaviors suggested in the teachers' editions of each textbook. The teacher behaviors suggested most frequently were: (a) explain using a specific example, (b) illustrate using the text display, (c) work through examples with students, and (d) elicit general student responses. Sixty-four percent of the lessons suggested questions for which the answer could be found directly in the lesson. It

should also be noted that research over the years has revealed that about 80 percent of the time class discussions are dominated by low-level questions, which is not surprising in light of these textbook findings.

Two approaches to geometry instruction, one that provided informal instruction followed by proof during the second part of the course and another that emphasized proof throughout, were compared by Han (April, 1987). The subjects of the study were 478 geometry students in two high schools, one of which used the first approach (informal followed by proof) and the other which used the second approach (proof throughout). A significant difference was found in favor of the second method in both proof-writing achievement and attitude toward proofs.

The use of textbooks by Saxon has been controversial for years in the mathematics education community. Saxon calls one of the features of his textbooks "incremental development." This refers to repeated practice and drill over extended time periods. Johnson and Smith (November/December, 1987) compared the Saxon Algebra I approach with a traditional textbook approach using two large (136 and 140 students) groups. One group received instruction from the Saxon book and the other from a traditional textbook. Johnson and Smith found no significant differences between the two approaches in the cognitive domain. The Saxon book used with the experimental group was popular with students and teachers.

Supplementary Materials and Calculators. Scott (January, 1987) studied the perceived uses of mathematics materials, Szetela and Super (May, 1987) investigated the uses of calculators for problem solving in grade seven, Heath (November, 1987) considered microcomputers as well as calculators, and Wells (June, 1987) looked into the relationships between journal writing and achievement among elementary school children.

In the past, mathematics teachers have often maintained classroom instruction close to the textbook presentations. Supplementary materials were used more often in elementary than secondary classrooms. Scott surveyed the uses of supplementary materials in grades one through five in 1981 and 1984 in a large urban school district. In 1981, about 90 percent of the teachers of grade five used rulers, the most-used material. This usage was followed by protractors, compasses, thermometers, volume measures, geoboards, calculators, math balances, base ten blocks, and geoblocks. In 1984, the use of each of these had increased. The use of calculators rose from 27 percent of the teachers in 1981 to nearly 94 percent in 1984.

The use of calculators for instruction by grade five teachers of a large urban school district increased from 27 percent of the teachers in 1981 to nearly 94 percent of the teachers in 1984.

Szetela and Super conducted a study which added to the growing evidence supporting calculator usage in mathematics classes. Twenty-four seventh-grade classes in a large Canadian school district were taught problem-solving strategies by specially trained teachers. In 14 of these classes the instruction involved the use of calculators. The calculator classes scored significantly greater than the non-calculator groups on attitudes toward problem solving as well as achievement on paper-and-pencil computations.

A Canadian study showed that seventh graders using calculators scored significantly better than non-calculator classes on attitudes toward problem solving as well as paper-and-pencil computations.

Heath found significant differences in favor of a calculator group in computation and attitude toward mathematics over a non-calculator group at the eighth-grade level in Phoenix, Arizona. However, he found that low-achieving eighth-grade students who had access to calculators did not achieve greater than those who used only paper and pencil for computations.

Wells investigated the relationships of mathematics instruction on measurement and place value with journal writing for pupils of grades one through three. A total of 250 students participated in the study. Wells found that journal writing with mathematics instruction appeared to help those third-grade students who had lowest prior knowledge of place value and regrouping among third graders.

Computers. During 1987, a very significant number of studies was done related to computers. Research was conducted at nearly every grade level. Computer assisted instruction, Logo, and various uses of microcomputers were the topics of investigations.

One always hopes that a question will be researched sufficiently so that stable, definitive conclusions might be established. It would seem that research related to computers may be close to that stage.

Shanoski (April, 1987) analyzed the effectiveness of microcomputer-assisted versus traditional instruction of primary and intermediate grade students. Eighteen intact classes of students received drill-and-practice microcomputer-assisted instruction for a minimum of 20 minutes per week for 20 weeks. Fourteen intact classes were not administered the drill-and-practice sessions but did receive traditional instruction. The microcomputer classes performed as well as the traditional classes on standardized achievement measures after the treatment period.

Grady (April, 1987) studied the use of computers for teaching graphing, the metric system, and problem solving to students of grades six and eight. These topics were chosen because teachers had identified them as difficult to teach and because software that did not dwell excessively on drill and practice was available. Four computers were placed in each of the experimental classrooms for two months. Teachers used the computers in large-group instruction, and the students worked with the computers for 30 minutes every third day of classes. The control group performed paper-and-pencil work without computers. The homework was the same for all students. Grady concluded that the computer-based instruction had a significant impact on the students' achievements.

Logo, a popular elementary programming language, was studied by several researchers. Rieber (February, 1987) looked at Logo with second-grade students, Hamada (January, 1987) studied Logo with fourth-grade students, and Kelly, Kelly, and Miller (Winter, 1986/87) worked with Logo and fifth and sixth graders.

Rieber pointed out that Logo may be the "...the most significant educational software of the decade" (p. 12). However, he noted that Logo had not lived up to all of its promises for providing a highly significant educational experience for children. He explained that the Bank Street College Project in New York and the University of Israel Project had cast serious doubts concerning the educational benefits of Logo.

Rieber conducted a three month study with second-grade students to investigate the claims that experiences with Logo will improve problem-solving abilities of students and will also provide incidental mathematical skills and insights. Twenty-five second-grade children of an experimental group received approximately one hour of Logo programming experience each week. The teachers working with students

used a guided discovery approach. The control group consisted of 22 children from a different classroom in the same school district as the experimental children.

Two pre- and posttest measures were given students of both classes. There were no significant differences on the pretests so the two groups were considered comparable. The posttest measures significantly favored the Logo experimental group on both problem solving and incidental mathematical learning. Rieber concluded that his study strongly supported the educational claims for Logo in problem solving and incidental mathematical learning.

On the other hand, Hamada found in a study of 150 third-grade students and 128 fourth-grade students from two suburban New York school districts that there were no significant differences among third and fourth graders who had been exposed to Logo and those who had not. On problem-solving tests, there was no difference at the third-grade level between the two groups, but in fourth grade, the non-Logo children outperformed the Logo group.

Kelly, Kelly, and Miller approached their evaluation of Logo in a different way. They inquired as to whether or not working with Logo would enhance basic understandings of angles and distances by fifth- and sixth-grade children. The study was conducted over one school year with a second-year follow-up of fifth-grade students who continued with Logo in the sixth grade. Two classes at each of the two grade levels were involved in the study, one receiving Logo instruction and the other not receiving the instruction. Kelly, Kelly, and Miller concluded that the effects of the Logo instruction were not sufficiently strong to enable them to conclude that after one year of instruction, the students of the Logo group could use angle and distance concepts better than students of the non-Logo group. They cautioned (from experiences gained with the follow-up study) that an exposure over a longer period than one year may be necessary before concepts such as angle and distance stabilize sufficiently in children to make differences in their understandings and uses of these concepts.

Research related to the effectiveness of Logo in developing problem solving and geometrical skills in children was mixed. Nevertheless, Logo has provided exciting experiences for elementary teachers and children!

Computer-assisted instruction (CAI) was also the focus of several studies. Hessemer Stegemann (April, 1987) and Pflug (November, 1987) studied CAI in grade four, Coleman (October, 1987) considered CAI in grade five, and Mevarech and Ben-Artzi (Fall, 1987) focused their study upon CAI in grade six.

How CAI drill and practice affected motivation and achievement of fourth-grade children was studied by Hessemer Stegemann. A sample of 69 fourth graders was partitioned into "achievers" or "underachievers" as determined by a comprehensive examination of basic skills. The children were randomly assigned to one of three methods: (a) CAI multiplication drill and practice with a reward game (CAIm), (b) CAI multiplication drill and practice without a reward game (CAI) and (c) an equivalent multiplication paper-and-pencil drill and practice without a reward (Pap and pen). The experiment lasted one month.

When motivation was determined by the amount of time of participation, the CAI and CAIm students were more motivated than the Pap and pen students. When achievement was measured as the number of multiplication problems done correctly, the CAI students achieved greatest followed by the CAIm and Pap and pen students in that order. When achievement was measured by an achievement test, there were no significant differences among the three categories of students.

Pflug also studied fourth graders while studying multiplication. She compared three classes: (a) a CAI group drilled on multiplication facts using a computer; (b) a peer-tutored group who were drilled by classmates on multiplication facts using flashcards; and (c) a control group who drilled independently. Following the administration of a multiplication fact test given to all three groups at the conclusion of the 25-day experiment, Pflug found no significant differences among the three groups.

In a grade five study, Coleman found that variations in the pace of CAI instruction did not appear to affect the response rate and accuracy of students on mathematical exercises.

Mevarech and Ben-Artzi studied 245 sixth-grade students in three Israeli schools. One school of 71 students had no CAI and was the control group. The remaining two schools were used for CAI work; one with 82 students employed CAI with fixed feedback and the other with 92 students used CAI with adaptive feedback.

All three schools used a common basic group instruction. The CAI schools had three periods of traditional instruction

and two 20-minute sessions of drill and practice at the computer. The school without CAI had four weekly periods of traditional instruction.

In CAI with fixed feedback, the students answered a random mixture of problems. The system constantly adjusted the level of practice to the ability levels of the students. The responses of the student were followed by a message on the monitor. The messages were "very good, good, or correct" depending upon the student's first, second, or third trial. For incorrect responses, the message was "you made a mistake." After three failed attempts, the correct result was given to the student.

The CAI with adaptive feedback was similar to that described above except that when a student answered exercises that were found to raise a student's anxiety levels, the responses would be "superb job or very fine work." For incorrect responses the messages were "think or try again."

Following the experimental period, the students were tested with measures of mathematics anxiety and mathematical achievement. The analyses revealed no significant differences on achievement measures, but CAI students tended to manifest lower levels of anxiety on "worries about mathematics" and "attitudes toward learning mathematics with computers."

Performance of students using CAI for drill and practice was usually no better than traditional instruction, but the students appeared to have less anxiety.

Jansson, Williams, and Collens (May, 1987) investigated the claim often made that experiences with programming will improve a student's logical reasoning skills. They concentrated upon studying conditional reasoning abilities related to four principles: (a) detachment or modus ponens, (b) inversion, (c) conversion, and (d) contraposition, and measured them with a 32-item test that contained eight items for each of the four principles.

Three groups were included in the study: (a) junior high school students; (b) students in a university teacher training program; and (c) university students enrolled in arts and sciences. In each of these groups were students who had not had any programming experiences. These students were selected for special programming

experiences. As a consequence of the study, the researchers could not support the claim that programming experiences improved conditional reasoning skills in students.

Computers in high school were studied by five investigators. Donnelly (February, 1987) concentrated upon grade 10, Gesshel-Green (August, 1987) focused upon students in Algebra II, McCoy (November, 1987) considered 800 high school students, Cooke (February, 1987) investigated college preparatory students, and Johnson (January, 1987) studied 30 profoundly deaf high school students.

Donnelly tested 177 tenth-grade students with the California Arithmetic Test and a mathematics anxiety rating scale before and after the experimental group received nine weeks of microcomputer instruction. The control group received no computer instruction. Donnelly found few differences between the two groups at the conclusion of the experimental period.

Gesshel-Green also studied the microcomputer with students. In this case, two intact Algebra II classes were used, with one class assigned to a treatment that included a microcomputer graphics program during class time and seven sessions in a computer laboratory. Using an Algebra II achievement test with both classes at the conclusion of the experiment, he found few differences between the two treatments.

Problem-solving skills including general strategy, planning, logical thinking, various algebraic variables, and debugging were identified by McCoy as common elements of computer programming and mathematical problem solving. She hypothesized that the programming experiences of students would transfer to an improvement in mathematical problem solving. After testing 800 high school students in Virginia after they had had programming experiences, McCoy found that the ability of students had the largest effect, whereas, programming appeared to have a moderate effect upon mathematical problem solving. She recommended that a curriculum should include computer programming experiences to improve the problem-solving abilities of students.

Cooke developed a software unit on exponential and logarithmic functions at the pre-calculus level. He tested the material with 11 randomly selected classes. The results were positive, indicating students could learn successfully from the software materials.

Johnson evaluated the effectiveness of two methods of instruction: (a) teacher-directed and (b) computer-assisted, for teaching abstract mathematical concepts to deaf students. A sample of 30 profoundly deaf students from four northern Illinois high schools participated in the study. The students were randomly assigned to the teacher-directed and

the computer-assisted groups. Students were taught in small groups of one to three students each. "Geometry with Logo" was used with the computer-assisted group, and the teacher attempted to duplicate this program with the teacher-directed class.

At the end of the instructional period, Johnson tested both groups with a geometry test, an abstract reasoning test, and a differential aptitude test. Johnson found no significant differences between the two modes of instruction.

Research has not indicated that students who were exposed to experiences in computer programming or computer-assisted instruction became better mathematical achievers or problem solvers than students who received conventional teacher-directed instruction. By the same token the computer experiences didn't seem to hinder their progress!

SUMMARY

We have reviewed research reported in 1987 related to the structure and organization of content, methods of content instruction, and instructional materials in mathematics. The relationship of an elementary school program to the instructional objectives of its district was investigated by Hughes. She found that the mathematics program was meeting the district objectives in early grades but not later grades. The reasons for this discrepancy are important. One cannot help but wonder if very many schools have this deficiency.

Avesar and Dickerson found that a child's use of the concept of one-to-one correspondence was highly dependent upon the perceptual system presented to the child. Windows, for example, instead of lines connecting elements failed to trigger one-to-one strategies in some children. The commonly asked question as to whether or not Algebra II should follow Geometry as a course in high school was researched, but no definitive conclusions were reached. A fascinating research project completed by Miura raised the question as to whether or not the structure of Asian languages having Chinese roots might influence the mathematical learning of Asian children because of the close affinity of these languages to basic number structure. She concluded that this affinity probably did

increase achievements. Classroom language which included uncertainty or bluffing was found to affect negatively the attitudes of students.

Counting is certainly one of the earliest number tasks learned by children. Baroody was surprised to find that a significant number of children had to be shown concrete counting strategies in order to perform simple addition tasks. Mitchell varied the counting situation by introducing the concept of "owing" candies. This novel approach was promising but not a panacea for problems of early additive number development. Finally, Adey explored the possibility that the actual task demand may so mask the conceptual situation for children that they fail to employ counting methods for addition situations. This is certainly worthy of future research attention.

The appearance of the equals sign in an exercise is often a signal to the student for "action" rather than "meaning," according to Cobb. The student seeing 3 + 5 = _____ is poised to act, probably because his or her teacher has stressed that interpretation. Mathematics educators hope this signal for action will be replaced by a relational concept, e.g., "3 + 5 is the same number as 8." Comiti and Bessot found that number guessing games operationalized as well as deepened number properties with French children. Greer found that decimals provided conceptual problems in 12- and 13-year-olds in Northern Ireland. For example, more than half of the children had difficulty selecting 0.62 as the largest number in the set: 0.62, 0.236, 0.4.

The relationships of Piagetian stages of cognitive development to van Hiele levels of geometric thought were investigated. Perhaps, not surprisingly, the relationships between the two were not stable in most students. Huber and Huber found that Austrian students were able to apply formal principles of probability, when confronted with problem situations. This finding may be counter to the current, popular contention that probability should be introduced from an exploratory data approach. It would seem that comparing the formal and exploratory data approaches to learning probability would be a fruitful area of future research.

Textbooks play an important role in mathematics instruction; yet, Flanders found that only about 30 percent of the material in a typical eighth-grade textbook was new, whereas, 88 percent of the algebra text was new. This represents an abrupt change for students! Research by Han seemed to indicate that delaying formal proofs in geometry textbooks detracts from students' abilities to actually do proofs.

Computers were the subject of numerous investigations reported in 1987. Overall, experiences in computer programming or computer-

assisted instruction did not significantly improve the achievements of students. It is probably time for mathematics educators to simply accept computers as highly useful tools that can enhance mathematical instruction rather than produce nebulous, hoped for by-products in students.

III. INDIVIDUAL DIFFERENCES, EVALUATION, AND LEARNING THEORY

Individual differences, evaluation, and learning theory are broad areas of research interest. Each is important to the total success of the teaching enterprise in mathematics. We must focus our teaching efforts upon individuals, we must evaluate the effectiveness of instruction and the progress of pupils, and we must have a theory of learning to guide our instructional activities. Analyzing student errors in computation is still important in mathematics instruction in spite of the availability of hand-held calculators. It is not the error, per se, that is important but rather the conceptual misunderstanding of the child that that error represents.

Mathematical anxiety continued to be of considerable research interest in 1987, and the popular question of searching for differences between sexes flourished. Researchers continued to find that mathematical anxiety is a multifaceted problem; consequently, the treatment of this problem must be complex. Research continued to support the contention that there are differences between boys and girls in learning mathematics, but the fundamental reasons remain elusive.

Evaluation is a many sided problem. It includes evaluation of students' progress, identification of gifted and less able students, evaluation of the instructional activities of teachers in the classrooms, and the complex problems of international comparisons. The United States appears to be losing the international race in mathematics instruction. We should be deeply concerned, but should we be alarmed?

Studies in learning theory appeared to emphasize the brain-computer analogy. The use of words such as coding, retrieving, long-term memory, metacomponential processing, schema acquisition, and rule automation is popular in this research area.

Individual Differences

In this section on individual differences, we shall review research related to analyses of errors made by elementary and more advanced students; research on low achievers and disabled students, and the ways in which small group instruction may be used in special circumstances; and finally research related to mathematical anxiety and sex differences in learning mathematics.

Analyses of Errors. McDonald, Beal, and Ayers (Fall, 1987/Winter 1987-88) studied the use of the computer to assist in the diagnosis

of addition computational skills in children; Taira (December, 1987) considered error reduction strategies for whole number operations with fourth graders; Chang (February, 1987) investigated the use of standardized test data for modifications of instruction; and Movshovitz-Hadar, Zaslavsky, and Inbar (January, 1987) and Hart (March, 1987) concentrated upon error analyses of the work of high school students.

McDonald, Beal, and Ayers analyzed possible computational errors of elementary school children by ten levels of problem description (e.g., level 8--3 digit + 2 digit with no renaming) and five major categories (e.g., basic fact errors). Using the ten levels of problem description and five categories, they constructed a 50-cell matrix. They also developed a 50-item addition test which was administered to 51 elementary students who took the test using microcomputers. The students used pencil and paper to work the problems and then entered the answers on the microcomputer. As a result, a bank of 2,550 problems was made available which contained 554 errors.

The errors were classified into five categories: operational, inversion, algorithm, basic fact, and unidentified. The largest category was "algorithm," which constituted 44 percent of the errors. The researchers described "inversion" as an error category that was not previously reported in the literature. Inversion consisted of the student inverting the order of digits in the entire sum (e.g., 175 for 571) or only inverting the units and tens digit (e.g., 517 for 571).

The researchers concluded that the computer can be an effective tool in addition error analyses, but that the analyses were totally dependent upon the answers the students put into the computer. They also noted that the computer analyses helped them discover that inversion was a "new" error pattern and, finally, that some errors were related specifically to the use of microcomputers (e.g., recording 4442 for 42, because of the "auto-repeat" function, when a key is held down too long.

Taira taught functional diagnosis, a diagnostic and corrective instructional approach, to selected fourth-grade students. She found that the students made fewer errors in addition, subtraction, and multiplication than students not taught this process.

Standardized tests are frequently given to students, and little is done with the results. Chang analyzed standardized test results for 373 seventh-grade students on the Comprehensive Test for Basic Skills. Diagnostic information was made available for specific classes. The information provided an analysis of

specific class weaknesses and then related these weaknesses to the textbook. It was found that this analysis helped teachers reduce student problems on concepts and applications but not on computation. Perhaps, weaknesses in computation were too well established by the seventh grade to be remedied effectively in this way.

Hart (March, 1987) described the British research project, Strategies and Errors in Secondary Mathematics (SESM), which was an outgrowth of an earlier project, Concepts in Secondary Mathematics and Science (CSMS), completed in 1979. CSMS had formulated 11 hierarchies of topics commonly taught in secondary schools. SESM focused upon the results that CSMS had investigated in algebra, fractions, and ratio and proportion. In particular, SESM selected from the CSMS data the erroneous answers and errors in the three areas of algebra, fractions, and ratio and proportion.

In algebra, SESM focused upon the meanings children gave letters in arithmetic; in fractions, it concentrated upon the interpretation of the division sign, the use of equivalent fractions in addition, and the number of fractions between two given fractions; and in ratio and proportion, it studied students' uses of ratio and proportion in problem-solving situations.

In algebra, it was found that students had difficulty with letters to represent numbers. In fractions, it was found that children do not conceive of fractions as an extension of natural numbers, that the "part of the whole" diagram was not a helpful model for operations on fractions, and that the notion of equivalence needed greater stress. In ratio and proportion, it was found that children would frequently "add" rather than "multiply" in applying proportional reasoning. SESM devised intervention strategies for each of these errors which were found effective in correcting them in children.

Movshovitz-Hadar, Zaslavsky, and Inbar analyzed the errors committed on the high school graduation examinations given in Israeli high schools. The analysis yielded six error categories: (a) misused data (e.g., adding to the problem data that was extraneous or irrelevant); (b) misinterpreted language (e.g., incorrectly translating natural language into mathematical language); (c) logically invalid inference (e.g., concluding from a conditional statement its converse); (d) distorted theorem or definition (e.g., applying the distributive property to a nondistributive function, $\sin (a + b) = \sin a + \sin b$); (e) unverified solution (e.g., steps were correct but the final result was in error); and (f) technical errors (e.g., $7 \times 8 = 54$). The researchers noted that distorted theorem or definition was the largest category of error, but that

one should not conclude that students made this error most often, but rather that the tests probably provided the greatest opportunity for this type of error.

Low Achievers, Disabled Students, and Small Group Instruction. Low achievers in mathematics were studied by the following researchers: Greene (February, 1987), Mika (August, 1987), Strosnider (August, 1987), and Williams (July, 1987). Greene sought to find explanations for poor multiplication and division fact memory in low-achieving fourth-grade students. He compared three treatments for instructing students about 15 multiplication and 15 division facts which they had previously failed. The three treatments were: (a) verbal rehearsal/visual imagery mnemonics instruction; (b) visual imagery/ verbal rehearsal mnemonics instruction; and (c) conventional or control group instruction. Greene found that mnemonics instruction significantly improved the experimental groups' performance in recall of multiplication and division facts as compared to the control group. Furthermore, it was found that visual imagery was superior to verbal rehearsal mnemonics instruction in recalling multiplication facts.

Instead of mnemonic instruction, Mika tried a mastery learning computer program approach with 96 remedial students in grades three, four, and five. She collected achievement data at two-hour intervals for ten hours and compared the three grades at each time interval. She found differences between the high- and low-achieving students within grades four and five at each time interval in achievement gains, but no differences between the two groups were found in grade three. Using a trend analysis for learning rates, she found no differences between comparison groups (high and low achievers) at any time. She concluded that the experiment did not support the contention that mastery learning would close the achievement gap between low and high achievers. She offered a possible explanation that the computer program may not have been a valid example of mastery learning.

Williams compared the effects of "tutoring" and "counseling with tutoring" of remedial mathematics students in the tenth grade. She pre- and posttested the students of experimental and control groups for achievement, self-esteem, motivation, and attitude. After three years, she found that tutoring combined with counseling significantly improved the achievement of students over tutoring only. Very surprisingly, self-esteem, motivation, and attitude did not differ significantly between the experimental and control groups. Williams felt that tutoring with counseling did have positive effects on the achievements of the remedial students.

Tutoring and counseling of remedial tenth-grade students improved their achievements over a three-year period, when compared with students who were only tutored.

Strosnider studied 270 handicapped ninth-grade students who had failed the Maryland Functional Mathematics Test in the fall of 1985 and were retested in the spring of 1986. She partitioned the retested students into two groups: (a) those who had passed and (b) those who had failed the retest. Data were collected concerning the students' general abilities, mathematical abilities, races, sexes, ages, attendance records, handicapped conditions, levels of special education services received, mathematics grades, and types of assistance received. It was found that general abilities, handicapped conditions, sexes, mathematics abilities, mathematics grades, and races of students in conjunction with each other explained success on the retesting of the Maryland Test.

Disabled students were studied by Dowell (February, 1987) and Fremder (May, 1987). Dowell pointed out that research had shown that children suffering common disabilities often exhibit common arithmetic error patterns. He classified 57 children of 8-16 years of age with a history of a learning disabled (LD) or cranial radiation therapy (CRT) into four categories: (a) arithmetic impaired (ALD), (b) reading and arithmetic impaired (RALD), (c) arithmetic impaired (ACRT), and (d) not subtype A (NCRT). After an analysis of arithmetic error patterns, he found no significant group profile differences in the four groups.

Rather than analyze error patterns, Fremder attempted to determine whether training visual dot pattern strategies in learning disabled students would transfer to other visual pattern tasks as well as to arithmetic sequencing. She included 42 learning disabled students in her study. These students were classified as neurologically or perceptually impaired in accordance with New Jersey standards. The 42 children were placed into three classes: (a) standard training; (b) standard training plus linguistic programming; and (c) a control group. The treatments consisted of three 20-minute sessions every second week for six weeks. The results of the study revealed significant transfer effects for the two treatment groups, but no differences between the two groups.

Grouping students, usually by means of measures of ability and/or mathematical achievement, has often been used to improve

the work of students, particularly less capable children. Grouping was the focus of attention in a study by Gerleman (September, 1987). On the other hand, Anderson (November, 1987) developed a model of mathematics education based upon "living systems" which involved grouping of students, too.

Gerleman observed 11 fourth-grade teachers who were chosen for study because they had indicated in an earlier survey that they used grouping methods for teaching the entire mathematics lesson during four or five days a week. Gerleman focused upon behaviors of teachers and students as they interacted in small groups. She also noted the teaching that took place, the tasks and assignments required of the students, interactions among students and teacher, how time was used, and how groups were organized and utilized. The 48 observations were made over an 11-week period during the second semester of the school year. The teachers were also interviewed to determine their motivations for using certain grouping procedures.

Gerleman provided detailed descriptions of her observations, which revealed the following: (a) students were usually divided into two or three groups, usually on the basis of achievement or ability, but most often on achievement, (b) the pace and content of instruction were usually different for each group, (c) the mean time allocated to mathematics was 46.4 minutes, with a range of 34.6 to 70.3 minutes, (d) in general, teachers worked with each group in turn, while the remaining groups did seat work, (e) there was no clear pattern for the amount of time that teachers spent with each group, with some teachers spending the greatest time with the high groups and others with the low groups, (f) teacher-group interaction usually consisted of checking homework, discussing errors, presenting a short lesson on new material, and review followed by controlled practice, (g) little time was spent on development of concepts and most time was devoted to review and practice, and (h) most teachers were quite adept at getting groups on task quickly, including the groups that they had planned to meet first.

Gerleman rated most of the teacher presentations low because she felt that teachers emphasized memorizing steps in algorithms rather than the reasons why algorithms work. Teachers often presented materials as review when the development of concepts should have been emphasized. Presentations usually consisted of one or two examples worked by the teacher or a student followed by practice. The instruction was usually teacher-directed.

Observations of elementary classrooms revealed that teachers emphasized the memorization of steps of algorithms rather than explanations as to why the algorithms worked.

In interviews with teachers, Gerleman learned that they usually used grouping approaches because the range of abilities was too great to teach the class as a whole. Few teachers had had any previous instruction in the use of grouping for teaching mathematics, although some had had instruction in grouping for teaching reading. Some teachers had used grouping procedures from the very first year of their teaching careers, while others had adopted this technique at later times.

Anderson developed a "living systems" metaphor embedded in the context of a small enrichment group in fourth-grade mathematics. During a two and one-half months period, the researcher observed the regular classroom and developed the "living systems" method in a small subgroup of six children. Video tapes and journal records were maintained during the entire period. From these findings, it was concluded that the "living systems" metaphor was actualized. Specific findings related to "boundaries", "heterogeneity," and other characteristics of living organisms were made. Overall, it was concluded that the "living systems" metaphor was a viable framework for the development of an alternative mathematics curriculum in grade four.

Mathematical Anxiety and Gender Differences. Mathematical anxiety and differences between sexes in learning mathematics have often been linked together in research studies. We shall review research on mathematics anxiety completed by Cemen (September, 1987) and Gliner (February, 1987).

Cemen provided a comprehensive description of the nature of mathematics anxiety by sythesizing general and test anxiety literature applied to mathematics, the mathematics anxiety literature, and interviews with students. Cemen developed a model of anxiety which includes environmental, dispositional, and situational antecedents. She claimed that anxiety reactions are a result of interactions of these antecedents. Students make decisions about dealing with anxiety. If self-esteem is strong and there is task-related confidence, the student may be able to channel the anxiety into the task, and the anxiety might help performance. However, if the person cannot control anxiety, it will hinder performance. Students may also cope with anxiety by avoiding mathematics or convincing themselves that it is not useful.

Gliner tested 95 students, 50 boys and 45 girls, in grades nine through 12 at an urban high school in Denver, Colorado. The Mathematics Anxiety Rating Scale-A (MARS-A) was administered to all the students. The students were enrolled in courses ranging from general mathematics through analytical geometry. In addition to the MARS-A data, a number of other variables were considered including sex, grade, age, course, number of mathematics courses, grade point average, spelling, language, and various mathematics test measures. Correlation coefficients were determined and a step-wise regression analysis was performed.

The correlational analysis revealed that all variables except sex, grade, and mathematics correlated negatively with the MARS-A scores. The correlation coefficient of greatest magnitude with the MARS-A scores was grade point average ($r = -.26$).

The step-wise regression analysis showed that grade point average, mathematics course, sex, spelling, and language expression contributed most to the variance in MARS-A scores. Furthermore, it was found that verbal skills were good predictors of overall mathematics achievement. This study confirmed the fact that mathematics anxiety is a multi-faceted phenomenon which is extremely difficult to define very precisely.

Sex differences in mathematics have been examined in two ways. The most prevalent of these is to analyze test results for females and males and then make comparisons of the two sets. The second way is to observe the participation of males and females in mathematical activities and compare their rates of participation. We shall begin with the latter approach by examining two studies, one by McLain (June, 1987) and another by Marshall and Smith (December, 1987).

McLain surveyed 72 western Michigan school districts to ascertain the effects that graduation requirements in mathematics had upon enrollments of males and females in mathematics courses. In particular, comparisons were made between school districts requiring one year and those requiring two years of mathematics for graduation.

McLain found that schools that required two years of mathematics had a higher proportion of eleventh- and twelfth-grade females enrolled in mathematics courses than schools that only required one year of mathematics for graduation. Furthermore, the survey also demonstrated that schools requiring two years of mathematics had significantly higher ratios of female-to-male students enrolled in advanced elective courses in mathematics than schools requiring only one year of mathematics. During

interviews with 316 algebra students in nine schools, McLain discovered that female algebra students in the ninth grade had significantly higher levels of anxiety than similar students attending schools requiring only one year of mathematics.

A survey of 72 western Michigan school districts found that a higher proportion of eleventh- and twelfth-grade females were enrolled in mathematics courses in schools requiring two years of mathematics for graduation than schools requiring only one year of mathematics.

Marshall and Smith conducted a longitudinal study of errors made by children at the third- and sixth-grade levels. A major finding of the study was that girls in the third grade performed better than boys in the same grade, but that this advantage seemed to disappear by the sixth grade. An explanation that they offered for the difference was that girls, because of biological or social/cultural conditions, appeared to have a greater capacity for sitting still and for performing tasks requiring small muscle coordination than boys of the third grade. Perhaps, these differences disappeared by the sixth year.

In addition, Marshall and Smith found significant sex differences in three categories: spatial understanding, associations, and erroneous rules. They advised approaching the spatial understanding advantage for boys with caution because of the limited number of observations. Observed differences on errors of association and erroneous rules were more stable. It was found that girls were more likely to make these errors at both the third- and sixth-grade levels than boys. The researchers strongly recommended that more research focus upon determining error differences between males and females on specific mathematical tasks rather than upon comparing overall test scores for the two sexes.

Numerous investigations on gender differences revealed by testing were reported in 1987. These studies included: McConeghy (April, 1987), Brandon, Newton, and Hammond (Fall, 1987), Moore and Smith (January, 1987), Dorans and Livingston (Spring, 1987), Feliciano (May, 1987), and Doolittle and Cleary (Summer, 1987).

McConeghy concentrated upon examining attitudes toward mathematics and achievement in mathematics as measured by the 1977-78 and the 1981-82 mathematical assessments conducted by the National

Assessment of Educational Program (NAEP). In these assessments, attitudes toward mathematics were measured by 14 statements which were used to construct three attitude scales and an overall attitude index. Analyses by McConeghy revealed that gender differences had the least influence upon attitudes and achievement. Not surprisingly, parents' education and the race of the student had the strongest influences.

Brandon, Newton, Hammond concentrated upon gender differences found in students of Hawaii. The Hawaii State Department of Education conducts annual statewide testings using the Stanford Achievement Test Series. In particular Brandon, Newton, and Hammond studied the results of the 1982-83 and 1983-84 testings for grades four, six, eight, and ten. Only data on four major ethnic groups (Caucasians, Filipinos, Hawaiians, and Japanese) were examined.

The results of the study showed that the public school girls in Hawaii had higher achievement levels than the boys in mathematics. The girls achieved their highest scores in computation, whereas, the the boys scored highest in mathematical reasoning. The high achieving females outperformed the high achieving males. Sex differences in mathematics favoring girls among Caucasian students were less than those favoring girls among Japanese-American, Filipino-American, and Hawaiian students. The researchers concluded that if these differences continued into adulthood, then 57 percent of the females would pass civil service tests given in Hawaii but only 43 percent of the boys would pass.

Moore and Smith studied mathematics achievement data of 11,914 people, aged 15 to 22, who were members of the National Longitudinal Study of Youth Labor Force Behavior conducted in 1979 through 1981. The test data of the study were obtained in 1981. Moore and Smith reported that general patterns showed that males outperformed females in mathematical knowledge and arithmetic reasoning. However, when performances of those who completed Grades K to 8 were examined, females outperformed males in mathematical knowledge. There were no differences between the sexes in arithmetical reasoning. When data of those who completed Grades 9 to 11 were examined, sex differences in favor of males appeared and increased with further education.

Dorans and Livingston noted that J. C. Stanley of the Study of Mathematically Precocious Youth had observed that nearly-all females who scored extremely high on the Scholastic Aptitude Test-Mathematical (SAT-M) did so because they were very strong verbally, whereas, some males scored extremely high on this test in spite of relatively low verbal skills. Dorans and Livingston decided to investigate further this observation.

They sampled students who had taken the SAT tests either in June, 1981 or in May, 1982, and included only those students whose SAT-M scores were at least 600 and had indicated that English was their best language. The June 1981 sample consisted of 13,761 males and 7,001 females, and the May 1982 sample contained 11,573 males and 6,819 females. The mean verbal scores of males tended to be lower than the mean verbal scores of females, when both groups had the same mathematical score. There was one possible exception to this finding for students who had achieved perfect scores of 800 on the SAT-M. There were four females with perfect SAT-M scores and each had extremely high SAT-V scores; and there were 95 males with perfect SAT-M scores whose verbal scores ranged from 420 to 780. The authors felt that their data partially supported the Stanley hypothesis.

It has been hypothesized that females who score high on the SAT-M Test do so because they have very high verbal skills, whereas, some males score high on this test in spite of low verbal scores. A study by Dorans and Livingston provided limited support for this hypothesis.

In a study similar to the one by Dorans and Livingston, Doolittle and Cleary sampled high school seniors who had taken the ACT Assessment Mathematics Usage Test (ACTM) in October, 1985. They found that males tended to perform better than females on geometric and mathematical reasoning items, whereas, females performed better than males on algorithmic, computation-oriented items. They suggested that, perhaps, males have developed relatively stronger spatial and reasoning skills than females, and that females have developed stronger algorithmic or computational skills. The reasons for these differences probably involved a wide range of cognitive and sociocultural factors, suggesting that extensive research on these factors is needed.

Felciano studied 1,000 randomly selected tests of sixth grade students from each of three administrations of the Puerto Rican Basic Skills Test in Mathematics-6. Felciano was interested in comparing female-male performances in problem solving. It was found that females outperformed males in problem solving and in most of the computational-type tests. An analysis of covariance was conducted to compare male and female problem-solving performance,

when computational skills were controlled. The results tended to support the finding that sex-rated differences did not exist for students of similar computational skill levels.

Evaluation

Research on evaluation reported in 1987 can be partitioned into four groups: (a) evaluation of instruction, (b) identification and prediction, (c) studies of instruments of evaluation, and (d) international comparisons. We shall begin with a review of research on evaluation of instruction.

Evaluation of Instruction. Three studies focused upon evaluation conditions and techniques vital to classroom instruction: Restaino (September, 1987), McDermott-Krieg (April, 1987), and Denvir and Brown (June, 1987).

Restaino investigated the effects of student participation in Chapter I programs during 1982-86. Standardized achievement measures in mathematics were given to one-year participants, two-year participants, and a control group. Restaino found that the data revealed the effects were positive, more so for two-year than for one-year participants.

McDermott-Krieg tested two groups of third-grade students using a second-grade mathematics test. One group took the test in a traditional paper-and-pencil mode, whereas, the second group was read all test information and directions via a paced tape recorder. She found no significant differences between the two modes of testing after studying the scores of the two groups on the second-grade mathematics test.

Denvir and Brown compared a classroom test assessment of 14 number skills with an interview assessment of the same skills. The skills included estimation, counting, and place value; addition and subtraction operations; and multiplication and division operations. The classroom test was given to 249 students, ranging in ages from nearly eight to nearly 12 years. A subsample of 32 children of the same ages were interviewed for their knowledge of the same skills as the classroom test.

The results of the comparison revealed that: (a) it was not possible to design test items for all the skills that were felt to be crucial in the understanding of number; (b) there were differences between students' performances on the same skill when the class test and interview results were compared; (c) of the 32 students interviewed, six would have been significantly misdiagnosed as they would have been incorrectly assessed on

at least 25 percent of the 14 skills by test results alone; and (d) the class assessment test did provide a valuable initial assessment as it was usually true that students who were successful on items of the class assessment were also successful on those same items during the interview assessment.

When comparing the two methods of assessment, it was found that the probing of students' understandings made possible by interviewing was crucial to a proper assessment of some skills. For example, in a question on calculator usage on the classroom test, a picture of the keys were presented to students, and they indicated which keys to press; whereas, in the interview, the actual calculator was made available to the child. This quite naturally produced differences in results.

When comparing a classroom assessment of number skills with an interview assessment of the same skills, it was found that the probing made possible through interviewing resulted in a superior understanding of students' number difficulties.

Five studies concentrated upon the evaluation of the teacher. These studies were done by Tredwell (February, 1987), Grace (May, 1987), Smith (September, 1987), Andrade (April, 1987), and Krampen (June, 1987).

Tredwell developed and tested a technique for observations of elementary mathematics instruction. She concentrated upon identifying, timing, and describing the manner in which teachers and students used time during mathematics instruction. The teacher behaviors included instruction, assessment, and administration. The student behaviors focused upon engaged and unengaged behaviors. The observational technique was field-tested in 50 elementary classrooms.

The results of the field testing indicated the method was feasible and reliable. In particular, it showed that the technique provided a reliable record of how teachers used class time, provided a description of teacher behaviors during mathematics instruction, and provided an accurate assessment of students' engaged behavior during instruction.

Grace concentrated upon identifying effective teacher behaviors for working with low-achieving students in beginning algebra classes.

Thirty-one classes of low-achieving algebra students were each observed for three consecutive lessons. The growth of the students in algebra achievement over the year was measured with the Cooperative Mathematics Algebra Test I.

Grace correlated teacher behaviors with achievement growth of students and concluded that effective teachers presented materials directly by explaining and demonstrating, asked many product and process questions, and provided all students many opportunities to respond. During seatwork, effective teachers circulated and assisted students less than ineffective teachers. Effective teachers also had fewer tardy and off-task students in their classes. With very low-achieving students, it was found that circulating and assisting them was needed for effective growth.

Teachers effective with low-achieving algebra students explained and demonstrated materials directly, asked many product and process questions, and provided all students many opportunities to respond to questions.

Models of classroom teaching have become popular during recent years. Smith evaluated anticipating set, objective and purpose, input, modeling, checking for understanding, guided practice, and independent practice for effectiveness with 34 teachers of grades one through six. Correlation coefficients were calculated between the use of each element of the model and students' mastery of mathematics, extent of teacher inservice, and teacher experience. Negative correlation coefficients were found between student mastery of mathematics and both input and modeling. Negative relationships were also found between years of experience and mastery of mathematics. Positive relationships were found between independent practice and mastery. Overall, it was concluded that the model, for the most part, was effective. Of course, strict adherence to any model can stifle spontaneity of instruction.

Andrade compared two groups of students in second through the fifth grades in Greeley, Colorado. One group was instructed by teachers who had nearly completed an essential elements of instruction program, and the other group was instructed by teachers who had just begun the training program. Not surprisingly, there were few significant differences noted in the two groups of students.

The effectiveness of writing comments on students' papers had been studied in the past, and the results were somewhat mixed. Krampen looked into this practice with teachers of Trier, Federal Republic of Germany. Krampen selected 13 teachers and 385 students of grades six through ten for the study. The teachers and students were randomly assigned to four groups, whose teachers made: (a) socially oriented comments on all mathematics examinations during the first semester of the school year; (b) subject matter comments during the same time period; (c) individually oriented comments during the semester; and (d) no written comments (the control group).

Following an extensive statistical analysis of attitude and cognitive variables involving interactions, Krampen concluded that: (a) socially oriented comments affected low-performing students negatively and high-performing students somewhat positively or not at all; (b) subject matter oriented comments affected all students with small positive gains; and (c) individually oriented comments showed low positive effects, with the low-performing students affected most. When comments were no longer made, the small positive effects disappeared.

Written comments made on examination papers concerning the subject matter of the examination had small, but positive, effects on most students.

Identification and Prediction. Research has dealt with factors for selecting gifted students, a comparison of prediction using tests or teacher recommendations, a measure of prediction for dyscalculia, prediction of students' success in Algebra I, and the effects of prior mathematical experiences on mathematics test scores.

DeRidder (May, 1987) partitioned 87 sixth-grade students into six groups using IQ scores and mathematics achievement. Based upon the Renzulli model for giftedness, the study assessed problem-solving abilities, creative mathematics abilities, and task commitment of the students in the six groups. DeRidder found that (a) a particular IQ range is inadequate for selecting gifted students; (b) a range of mathematics achievement could serve in the identification of gifted students; (c) students gifted in other subjects are not necessarily gifted in mathematics; and (d) giftedness is too complex to be measured by a single measure.

Research has shown that IQ scores alone are inadequate as a means of identifying gifted mathematics students in the sixth grade.

The selection of gifted sixth-grade students was also the subject of a study by Goodman (April, 1987). Goodman compared two methods of selection: (a) test scores; and (b) teacher observations. At the conclusion of the sixth-grade year, Goodman tested the students with a teacher-developed test of computation, geometry, and metric measurements. These test scores and observations were compared with the success of students in an accelerated seventh-grade mathematics class a year later. Goodman found that the test scores were better predictors of student success in the accelerated class than teacher observations.

Geiman (May, 1987) evaluated a sample of average seventh-grade students with a multidimensional mathematics test to determine whether or not the students could perform test items without symptoms attributed to dyscalculia. Twenty-two subtests of the test were based upon evaluations of dyscalculia currently used. The test may be of some use in predictions of dyscalculia, but the researcher advised caution in its use.

As Elgammal (October, 1987) had noted, the selection of students for Algebra I has been a dilemma for mathematics educators for many years. Testing 413 students in Detroit, Michigan, Elgammal collected data on nine variables and determined correlations among them. It was found that prior mathematical knowledge as measured by a test was the best single predictor of success in Algebra I. This predictor was followed in predictive effectiveness by an attitude measure.

Jones (May, 1987) analyzed data from 7,500 members of the 1980 sophomore class of the High School and Beyond Project. He found that senior-year mathematics scores were highly dependent upon number of mathematics courses studied, including Algebra I or above, and were reasonably well predicted from socioeconomic status, sophomore verbal test scores, and sophomore mathematics test scores. He also found that whether students were black or white, female or male, sophomores with similar achievement levels may be expected to experience similar improvements by studying additional mathematics courses. The improvement was especially pronounced for those students with four or more years of mathematics

or with three years that included calculus. Jones encouraged greater enrollment of black and female students in advanced high school mathematics courses.

Studies of Evaluation Instruments. Secolsky (March/April, 1987) studied minimum competency tests, Solomon (April, 1987) compared secondary school standardized mathematics achievement test questions with objectives of the National Assessment of Educational Progress (NAEP), and Grandy (October, 1987a) focused upon ten years of Scholastic Aptitude Test (SAT) scores and student plans for mathematical study.

Minimum competency tests have become popular nationwide. These tests generally dwell upon computation, simple geometry, and simple algebra. Secolsky had 99 high school mathematics teachers from eight New York City public high schools rate the difficulties of eight items from the New York State minimum competency test in mathematics. Four of the items were computational exercises and four were word problems. The teachers came from schools which had large numbers of lower ability students.

The teachers rated each item as: (1) too easy, (2) easy, (3) medium, (4) difficult, or (5) too difficult. Some examples of the ratings were: (a) "Subtract 974 from 4,009" was given a mean rating of 2.14; (b) "Solve for X: $7X - 4 = 24$" was given a mean rating of 3.01; and (c) "Divide 14.71 by 0.8. Round off your answer to the nearest hundredth" was mean-rated at 3.53.

A study of those teachers who rated competency test items as "easy" tended to: (a) favor tougher standards in mathematics; (b) graded students relative to other students; and (c) interpreted each item in terms of more general skills. Selecting judges to rate items involves considerations of attitudes and motivations of judges.

Solomon looked into the relationships between standardized test items and NAEP objectives and subobjectives. He had a panel of experts from the Philadelphia school district classify items from standardized mathematics tests according to NAEP objectives and subobjectives. The tests studied by the Philadelphia experts included: (a) California Achievement Tests, Levels 19 and 20; (b) Comprehensive Test of Basic Skills, Levels J and K; (c) Metropolitan Achievement Tests, Advanced 1 and 2; (d) SRA Survey of Basic Skills, Levels 36 and 37; and (e) Stanford Achievement Tests, Advanced, Task 1, and Task 2.

Solomon found that these standardized tests did not differ significantly from the NAEP objectives and subobjectives. All

tests addressed 82 percent of the objectives and 48 percent of the subobjectives. No individual test covered more than 65 percent of NAEP objectives or 20 percent of subobjectives.

Of 11 popular, standardized mathematics tests at the secondary school level studied by a panel of experts, not one covered more than 65 percent of NAEP objectives or 20 percent of NAEP subobjectives.

Grandy analyzed results of the SAT taken by high school seniors between 1975 and 1986. Grandy found that: (a) examinees planning to major in mathematics, science, and engineering in 1986 obtained SAT verbal scores that averaged 19 points higher than the mean of all examinees and had SAT mathematics scores that averaged 38 points higher than the average for all examinees; (b) there were more males than females interested in mathematics, science, and engineering; (c) women planning to major in electrical, mechanical, and civil engineering had higher SAT mathematics scores than men intending to major in those areas; and (d) the mean SAT mathematics scores of blacks planning to major in mathematics, science, and engineering had risen considerably from 1975 to 1986.

In studying SAT mathematics scores of tests administered during 1975 to 1986, Grandy found that women intending to major in electrical, mechanical, or civil engineering scored higher than men intending to major in those fields.

International Comparisons. As the world shrinks, comparisons among nations are inevitable. While we should strive to learn as much as we can by studying other countries, we should be careful not to emulate them merely because of international competitiveness. Each nation has its peculiarities and cultural characteristics that make it unique, and effective educational practices in one nation may not at all be effective in another.

International comparative testing was reported in 1987 by Horvath (May, 1987), Al-Mahmoud (May, 1987), Sano (April, 1987),

and Stevenson (Summer, 1987). Observations of classrooms in Japan, Taiwan, and the United States were provided by Stigler, Lee, and Stevenson (October, 1987), and cultural beliefs about children's mathematical performances in China, Chinese-American, and Caucasian-American families were reported by Hess, Chih-Mei, and McDevitt (June, 1987). Finally, Song and Ginsburg (October, 1987) studied the development of formal and informal thinking in Korean and American children.

Horvath discussed the results of the Second International Mathematics Study (SIMS) conducted in 1981-1982 (1980-1981 in Japan) by the International Association for the Evaluation of Educational Achievement (IEA). SIMS studied two populations: (a) students who were between 13 years and 13 years, 11 months and (b) students who were in the normally accepted terminal grade of secondary school. The populations were designated "A" and "B," respectively. There were a number of anchor items that were used in the First International Mathematics Study (FIMS) as well as SIMS. These anchor items were the primary focus of Horvath's study. Anchor items covered computation, comprehension, and applications.

Horvath found that students of the United States at the eighth-grade level (population A) were closer to their Japanese counterparts on anchor items than students of the two nations at the twelfth-grade level (population B), where the Japanese significantly outdistanced the American students. Furthermore, it appeared that the United States had improved in the teaching and learning of lower-order skills, but not much progress had been made with higher-order skills.

Horvath, who had studied the FIMS and SIMS results and had also researched Japanese high schools which included extensive observations in Japanese classrooms as well as interviews with Japanese students, teachers, and counselors, offered the following alarming conclusion:

> **"...I have become convinced that more than minor adjustments are needed if the level of American junior and senior high school mathematics achievement is to improve."**
>
> **Horvath, May, 1987, p. 367.**

Another comparison of twelfth-grade students in mathematics was reported by Al-Mahmoud. Al-Mahmoud found that twelfth-grade

American students scored higher than their Jordanian counterparts on the SAT-M test. On the other hand, Sano found that beginning ninth-grade Japanese students performed significantly higher than comparable American students on perceptual and reasoning tests. American students scored higher than the Japanese on a verbal reasoning test. Stevenson commented upon the relative slowness of American children to acquire mathematical skills, when compared with children of Japan, Taiwan, and China.

In a comparative study of mathematics classrooms in Japan, Taiwan, and the United States, Stigler, Lee, and Stevenson conducted observations in 20 first-grade and 20 fifth-grade classrooms in Minneapolis; Sendai, Japan; and Taipei, Taiwan. It was felt that those cities represented comparable urban areas in their respective countries. The observers of the classrooms used time-sampling methods in which they coded the presence or absence of predetermined conditions. In each city, 1600 hours of observations were made, 800 at the first-grade and 800 at the fifth-grade levels.

In all three cities, language arts and mathematics dominated the classroom time. In Minneapolis, 62 percent of the class time was spent in reading or mathematics, followed by 59 percent in Taiwan and 54 percent in Japan, but American teachers of both grade levels spent less time in mathematics than teachers in Taiwan or Japan. Fifth-grade American teachers spent less than half as much time in mathematics as in reading. The average number of hours per week spent in mathematics in American first grades was 2.9, which compared with 3.9 hours in Taiwan and 6 hours in Japan. Average figures for the fifth grades were 3.4, 11.4, and 7.6 for the United States, Taiwan, and Japan, respectively.

Children of all three cities spent time in the classroom on inappropriate activities; that is, off-task behaviors. The United States led in this category with 17 percent, followed by 10 percent of the time for both the Chinese and Japanese children. In Japan and Taiwan, teachers taught mathematics to the entire class for 86 and 77 percent of the time, compared to 46 percent for American teachers. American teachers spent greater amounts of time working with individual students.

It was also evident to the observers that Japanese and Taiwanese students and teachers were more intently involved than American teachers and students. Furthermore, Chinese and Japanese teachers appeared to be better prepared and conducted more lively and varied classes than the Americans.

After comparing first- and fifth-grade classrooms in the United States, Japan, and Taiwan, Stigler, Lee, and Stevenson wrote: "..., we must conclude that American children fail to receive sufficient instruction. They spend less time each year in school, less time each day in classes, less time in the school day in mathematics classes, and less time in each class receiving instruction."

Stigler, Lee, and Stevenson, October, 1987, p. 1284.

Hess, Chich-Mei, and McDevitt interviewed mothers of sixth-grade children in the People's Republic of China (PRC) and Chinese-American and Caucasian-American groups in the United States. The mothers were first asked to rate their children according to their performances in sixth-grade mathematics on a six-point scale ranging from (1) not doing as well as most children to (6) doing the very best in class. If the child was rated as doing well, the interviewer placed five cards before the mother with one of the following on each card:

(a) My child has natural ability for math.
(b) My child tries hard in math.
(c) My child has had good training at school in math.
(d) My child has had good training at home in math.
(e) My child has been lucky in math.

The mother was then given ten plastic tokens to distribute over the cards according to the importance of that attribute. On the other hand, if the mother indicated the child had not done well, the negatives of the five statements were given with similar instructions for placement of tokens.

Lack of effort was the predominant cause for poor performance given by mothers of PRC, with little responsibility given to other factors. The Chinese-American mothers held similar views but with less weight assigned to effort and more to lack of natural ability, poor school, and poor home training. The Caucasian-American mothers gave more weight to lack of effort than other factors, but distributed blame more evenly across all other factors than did the Chinese mothers. The Caucasian mothers assigned more weight to causes not under their control (e.g., child's ability, luck) than did the Chinese mothers.

Song and Ginsburg tested 315 Korean children and 538 American children in the four- to eight-year age levels with the Test of Early

Mathematics Ability (TEMA). TEMA contains 50 items, 23 designed to measure informal mathematics (activities that do not involve written symbolism) and 27 designed to measure formal mathematics (activities involving written, symbolic, school mathematics). The children were tested individually by trained interviewers.

Song and Ginsburg found that the performance level of American children in informal mathematics was higher than that of Korean children at the ages of four and five. By the age of eight, the Korean children outperformed the American. When formal mathematics was considered, the American and Korean children did equally well at ages four, five, and six; but at the age of seven, the Korean children surpassed the Americans. The superiority of the Korean children could be found in every subarea of formal mathematics by the age of eight. The American children were poor both in accuracy and procedures of calculation.

Song and Ginsburg concluded that the superior performance of the Korean children seemed to stem from an efficient educational system, knowledgeable teachers, involved parents, and a culture that is supportive of mathematical learning.

Learning Theory

We shall partition studies involving learning theory into two categories: (a) studies of cognitive factors that strongly influenced or drove teaching practices; and (b) pedagogical factors that influenced or changed cognition. Examples of the former are effort training to affect instruction, the interaction of affective and cognitive factors that change mathematical reasoning, and Piaget's concept of number related to third-grade arithmetic. Examples of the latter are varying reinforcement schedules to improve understanding, mastery learning to improve achievement, and sequencing examples and non-examples to achieve concept attainment.

Cognitive Factors That Influence Teaching Practices. Geary (January, 1987) studied the internal and external validation of cognitive arithmetic component processes, McNabb (January, 1987) investigated strategy and effort attribution training as it affects greater competence, Kaplan (May, 1987) studied mathematical thinking and instruction, Smieciuch (November, 1987) considered Piaget's concept of number in relation to third-grade arithmetic, Clements and Nastasi (October, 1987) focused upon measurement of metacomponential processing in young children, Cooper and Sweller (December, 1987) studied schema acquisition and rule automation on problem solving, O'Donnell and O'Donnel (October, 1987) looked at cognitive preferences and ethnicity, and Moore and Stanley (1987) investigated family backgrounds of Asian Americans who reasoned well mathematically.

Geary hypothesized that elementary component processes for cognitive arithmetic included encoding of integers, retrieving information from long-term memory, and carrying to the next column. Metacognitive processes included monitoring of performance and choosing and executing the performance component. One hundred subjects were given simple and complex addition and multiplication exercises followed by tests of number facility, perceptual speed, and spatial relations. A strong relationship was reported between speed of information retrieval, facility of execution, and traditional numerical facility.

McNabb tested 111 fifth and sixth graders to assess their perceived competencies in mathematics, their attributional tendencies, and their attitudes toward strategy and effort. Following this testing, one-half of the students received training in effort attribution and the other half training in strategy attribution. The 111 students were then administered 13 difficult mathematical problems. McNabb found that when the effects of student's attitudes toward strategy and effort were controlled, strategy attribution was more effective than effort attribution, particularly for subjects with high perceived competence in mathematics.

The development of mathematical thinking as a function of interactions between affective and cognitive factors was investigated by Kaplan. Kaplan videotaped teachers and children engaged in mathematical activities. The tapes suggested that different kinds of non-cognitive factors were associated with certain types of mathematical thinking. In particular, Kaplan was able to classify the children as either pro-mathematical thinkers or anti-mathematical thinkers.

The relationships of Piagetian tasks to performance in arithmetic has interested researchers for years. Smieciuch assessed 36 third-grade children on 12 Piagetian tasks and seven kinds of arithmetic problems. He found that performance on the Piagetian tasks of classification and seriation accounted for the most variance in children's abilities to assign numerals to sets. Performance on transitivity tasks accounted for the most variance in performance on multiplication and multiplication-division problems.

Clements and Nastasi tested a theory attributed to Sternberg concerning metacomponential processing. The following metacomponents were studied: (a) deciding on the nature of the problem, (b) selecting performance components relevant to the solution of the problem, (c) selecting a strategy for combining components of performance, (d) selecting a mental representation, (e) allocating resources for solution of the problem, (f) monitoring solution processes, and (g) being sensitive to external feedback. Forty children were audiotaped while pairs of them solved a problem-solving

task. The tapes were transcribed and individual statements were assigned to the seven metacomponents listed above.

Clements and Nastasi found that "being sensitive to feedback" occurred in about 13 percent of the statements, and "selecting a mental representation" did not occur. The percentages of occurrences of the components in the order listed above were 4.65, 5.73, 1.57, 0.00, 0.32, and 12.97, respectively. Clements and Nastasi concluded that the reliability and validity of the Sternberg observational approach were acceptable.

Cooper and Sweller investigated schema acquisition and rule automation on mathematical problem solving. They defined schema as a construct which permits students to group problems into categories such that each problem in the category requires a similar type of solution. They hypothesized that schema acquisition preceded rule automation and furthermore that the use of worked examples would facilitate transfer and performance on similar problems. They further hypothesized that extensive transfer would not occur until problem-solving operators had been automated.

Cooper and Sweller conducted four experiments to test the hypotheses. Experiment one involved 24 eighth-grade students from the most advanced mathematics class at a Sydney, Australia high school. The second experiment involved 104 eighth-grade students from the top classes in two Sydney high schools. Both experiments contained simple algebra transformational problems with the second experiment being a replication of the first. The students in both experiments were given explanation sheets with worked examples. Subjects were given an opportunity to ask questions. Following this, 12 students worked on examples in the conventional way and 12 worked problems in which the first of each set was written out with explanations. Cooper and Sweller found that the experimental group was better able to solve both similar and transfer problems than the conventional group. Experiment three, using verbal protocols, and experiment four, using algebra word problems, also supported the hypotheses. They concluded from these experiments that schemas and rule automation may help problem solving, that schema acquisition occurred before rule automation, and the the use of worked examples facilitated both of these constructs.

O'Donnel and O'Donnel tested 175 students (72 Hispanics, 48 Blacks, 33 Anglos, and 22 Orientals) with a cognitive preference test and a first-year algebra achievement test in two high schools of Denver, Colorado. They concluded that the results indicated: (a) Oriental students scored significantly higher than Blacks on graphics; (b) Hispanics had significantly higher verbal scores

than Orientals; and (c) Blacks had significantly higher symbolic scores than Hispanics.

Moore and Stanley administered a questionnaire concerning parents' and grandparents' educational backgrounds to 68 students of Asian descent who had scored in the 700-800 range of the SAT-M before the age of 13. These students were classified as mathematically precocious. Most were first-generation Americans and came from well-educated families.

Moore and Stanley concluded:

> **"The phenomenal mathematical precocity of these Asian Americans, especially females, provides a superb example of ability, ambition, and willingness to work hard that is helping set higher levels..."**
>
> **Moore and Stanley, ERIC: ED 282 367, 1987, p. 66.**

Pedagogical Factors that Influence Cognition. Research reported in 1987 in the category of pedagogical factors that influenced cognition included research on mastery learning, the use of examples and non-examples in concept attainment, the effects of reinforcement and feedback on learning, and learning related to spatial attributes of students.

Mastery learning, which permits students to move at their own pace, assuming that they demonstrate mastery of the materials as they proceed, was researched by Sullivan (September, 1987). The purpose of the research involving junior high school students enrolled in general, remedial, and resource mathematics was to compare outcomes of mastery learning and traditional learning. All students spent one semester in each treatment.

Sullivan found that the students in the mastery mathematics treatment scored significantly higher on tests of application than the students in traditional learning, but the traditional students outdistanced the mastery students on tests of computational skills over one semester. The original mastery learning group outscored the original traditional group at the end of the year in all measures. Discipline problems and equipment failure were cited by the teachers as disadvantages of the mastery learning method.

In another mastery learning study with 273 fifth-grade students in Oklahoma, Cox (July, 1987) compared measures of achievement, self-concept, and attitude of students in mastery learning and non-mastery learning situations. She found that the academic achievement of the high ability students of both groups differed, favoring mastery learning. There were no differences in the middle- and low-ability students of both treatments. No differences were found in the measures of self-concept and attitudes.

Many mathematics educators believe that students should experience both examples and non-examples in order to understand a concept, that is, one must see a "non-square" as well as a "square" in order to understand the concept of square. Petty and Jansson (March, 1987) compared the effects of using two strategies of sequencing examples and non-examples of parallelogram with Canadian sixth graders. The two sequences were rational and random. In the rational sequence, at least two pairs of matched examples and non-examples were presented in ascending order of difficulty. An example and a non-example are matched when their irrelevant attributes are similar. As one might expect, the rational sequence proved superior to the random sequence on a test of concept attainment of parallelogram.

Reinforcement of learning and effects of feedback were investigated by Pucci (February, 1987), Bonanno (February, 1987), Birenbaum and Tatsuoka (Summer, 1987), and Gallagher (April, 1987). Pucci found that randomly administered reinforcers in a computer-assisted mathematics task with 83 subjects produced the greatest positive effects. Bonanno learned that systematic teacher reinforcement containing either academic or self-esteem orientation was superior to neutral, unsystematic reinforcement with 36 learning disabled male high school students. Birenbaum and Tatsuoka compared three kinds of feedback: (a) information about the correctness of response, (b) the correct answer, and (c) the correct rule for problem solution. The effects of the feedback modes were differential and dependent upon the seriousness of errors. In general, the more feedback that was provided the more effective was instruction. Gallagher found that a delayed feedback of 45 seconds produced greater gains in correct responses with 50 sixth-grade students on a microcomputer drill and practice program on basic addition facts than immediate feedback program.

Spatial perception and spatial visual abilities were investigated by Del Grande (April, 1987). Del Grande inserted a strong geometrical component in a Canadian second-grade program. He developed a spatial perception test of 10 spatial categories and 40 items. Using a series of testings, Del Grande concluded that this geometrical intervention did improve the spatial perception of second-grade students.

Del Grande found that the introduction of a special geometrical unit in the second grade of some Canadian elementary schools substantially improved students' spatial perception abilities.

SUMMARY

We have reviewed research reported in 1987 in three broad areas: individual differences, evaluation, and learning theory. We shall summarize the reported research for each of these topics.

Individual differences consisted of research reviews on analyses of student errors; low achievers, disabled students, and small group instruction; and mathematical anxiety and sex differences. Analyses of student errors continued to be an examination of the steps of algorithmic processes performed by students when adding, subtracting, multiplying, and dividing numbers. Perhaps, the big change was brought on by the use of the computer in classifying error types. McDonald, Beal, and Ayers identified inversion of digits in the answer as an error type not previously reported. Hart's research on algebra errors found that often children incorrectly add rather than multiply in proportional reasoning problems.

Williams found that tutoring and counseling of remedial tenth-grade mathematics students proved to be an effective practice. Grouping is common in elementary reading instruction, but less common in mathematics teaching. Gerleman provided a detailed analysis of grouping practices of fourth-grade teachers without finding a clear cut pattern of activities employed by teachers. Such activities were largely a function of the teacher's style and attitude toward students.

Mathematical anxiety was of continued research interest, and the Mathematics Anxiety Rating (MARS) Scale was most often used to measure anxiety. The search for factors that promote anxiety did not produce any definitive results. Anxiety remained a multifaceted, complex problem. Research on sex differences continued to document differences found in testing, but the reasons for such differences remained an open question. The relationship of high mathematical abilities and high verbal skills for outstanding females but not necessarily for males was a promising hypothesis needing further study.

When evaluating number skills of students, the interview process, although time-consuming, remained a superior method of probing understandings when compared to paper-and-pencil testing. Evaluation of teachers in action in the classroom showed that teachers who entice student activity with questions and explanations produced excellent results. Identifying gifted sixth-grade students required measures in addition to IQ scores in research reported by DeRidder.

Solomon found that the NAEP objectives and subobjectives were tested by most standardized tests that were examined, and that no one test covered more than 65 percent of the NAEP adjectives.

International comparisons continued to intrigue researchers. Oriental children outperformed American children. Were the reasons for these differences educational, parental, cultural or a combination of factors? The answer to this question remained unclear. Much improvement is needed in all factors, if American mathematics education is to be truly competitive in the international mathematics education race.

Learning theory research appeared to lean heavily upon computer analogies. Words such as encoding, retrieving, long-term memory, metacomponential processing, schema acquisition, and rule automation were plentiful in this research. Research related to Piagetian tasks was still popular. Studies made of examples and non-examples in concept attainment showed that rational sequences were superior to random sequences in generating concept attainments.

IV. TEACHER EDUCATION

Teachers represent the heart of the educational enterprise, and their education is crucial to the vitality of education. Consequently, research in teacher education is important. Although a reasonable amount of activity was reported in 1987, the amount of research was not overwhelming. This research can be viewed in two categories: preservice teacher education and inservice teacher education.

In preservice education, the perceptions of preservice teachers toward mathematics and teaching are revealing. Unfortunately, the preservice teachers studied did not view their preservice training as highly useful to their careers. The preservice preparations of mathematics teachers were studied, the use of computers was investigated, and student teaching, the climax of teacher preparation, was examined by researchers.

The thoughts and perceptions of inservice teachers are also vital, perhaps, more so than those of preservice teachers. Inservice teachers frequently act out "scripts" in the classroom, when diagnosis and analysis were in order. Teachers and, perhaps, mathematics teachers to a greater extent, fall into routine patterns of teaching. Large classes, numerous reports, and rigorous work schedules often contribute heavily to such teaching behaviors.

Preservice Teacher Education

The research on preservice teacher education will be viewed from four perspectives: (a) perceptions that prospective teachers had about mathematics and teaching, (b) the preservice preparation of teachers in mathematics and pedagogy, (c) computers and preservice education of teachers, and (d) student teaching experiences.

Perceptions of Mathematics and Teaching. Owens (September, 1987) provided some alarming findings related to the perceptions of four preservice secondary mathematics teachers toward mathematics and teaching. Each of the four preservice teachers completed a series of seven one-hour interviews designed to determine their views related to mathematics and teaching.

The preservice teachers judged mathematics in terms of its perceived usefulness in the secondary curriculum and in terms of their successful experiences in secondary school mathematics. Their successes in precollege mathematics played an important role in shaping their perceptions of mathematics. They viewed mathematics in terms of their ease in learning it and the ease that they anticipated in teaching mathematics. Unfortunately,

preservice college mathematics courses did not present to them experiences that were compatible with their perceptions of mathematics. These courses were not viewed as being useful and relevant to their future positions as secondary school teachers.

Emotional constructs played a more significant role than practical or intellectual constructs in the four teachers' interpretations of their undergraduate experiences. College-level mathematics courses viewed against their backgrounds of secondary mathematics were not judged as being relevant or practical.

Owens' study is a serious indictment of the mathematical preparation of prospective teachers. One, of course, must view these findings in terms of the purposes of large, multipurpose universities. Mathematics courses in such institutions serve a wide range of students. Prospective teachers are only one of many kinds of students being taught, and their future needs in teaching can be easily misjudged or overlooked.

Evans (January, 1987) focused his study upon freshmen who were oriented toward teaching but were enrolled in a large Midwestern engineering school. These students had not chosen to enter teaching, and Evans attempted to determine their reasons for not selecting teaching. Obviously, the lure of an attractive career in engineering in addition to the compensation and prestige of engineering weighed heavily in their decisions. Evans discussed other reasons for their disinterest and suggested ways that these reasons may have been overcome. Certainly, it is important that mathematics teaching attract more and better teachers to the profession, and further research on this crucial question is needed.

Preparation in Mathematics and Pedagogy. Bitter and Cameron (Summer, 1987) discussed screening of preservice teachers for mathematical skills; Ginther, Pigge, and Gibney (November, 1987) concentrated upon the mathematical preparation of future elementary school teachers; Merrill (November, 1987) focused upon preservice teachers' understanding of division; and Trent (February, 1987) made a plea for better prepared junior high school mathematics teachers.

From January, 1983 to June, 1987, the microcomputer research clinic of the College of Education of Arizona State University gave the Computer-Administered Mathematics Examination 472 times to 205 students. The test, which had many computergenerated equivalent forms, was designed to identify preservice teachers who were weak and needed remediation in mathematics. Of the 205 students, 104 failed on their first attempts, and 61 of the 104 passed upon a retesting. Bitter and Cameron described four groupings of the 205 students. The Main Group was the 205 students, the

Pretest-Posttest Group consisted of 61 students of the 205 who had taken more than one test to pass, the Timed Group consisted of 91 students who had their responses timed, and the Timed Pretest-Posttest group consisted of 27 students who failed the first test and retook the test one or more times.

The primary finding for each of the groups was as follows. In the Main Group, 101 of the 205 students passed on the first attempt. There were increased scores for all students of the Pretest-Posttest Group, even though the "posttest" may have been taken as many as four times. The Timed Group required an average of nearly 74 minutes for the entire 50 items with a range of 30 to 152 minutes. In the Timed Pretest-Posttest Group, a mean gain of nearly 18 percentage points was achieved from the first to the last testings.

Ginther, Pigge, and Gibney gathered data in 1967-69 and 1983-85 concerning mathematics courses studied by prospective and inservice elementary teachers at Bowling Green State University, the University of Toledo, and Eastern Michigan University. Approximately 20 percent of the inservice and 80 percent of the preservice elementary teachers were also tested during 1967-69 and in 1983-85 with the same 65-item test. The test covered sets, numeration systems, fundamental arithmetic operations, number theory, and geometry.

The researchers found that the percent of elementary teachers taking four or more years of high school mathematics rose from 16 percent to 31 percent when 1967-69 results were compared to 1983-85 results. The mean gain on the test from 1967-69 to 1983-85 was 0.2, which was insignificant. The percent of elementary teachers completing more than three college-level mathematics courses rose from four percent to 23 percent in the same time interval. The mean loss on the mathematical test was 4.25. Consequently, elementary teachers of 1983-85 completed more college mathematics courses than teachers of 1967-69, but did not possess as much mathematical knowledge as those teachers.

Research revealed that the percentage of elementary teachers taking three or more years of college mathematics rose from four to 23 percent when data of 1967-69 were compared to data of 1983-85. Surprisingly, the mathematical understandings of the 1983-85 teachers were less developed than those of 1967-69!

Concept mapping was the focus of a study by Merrill (November, 1987). Concept mapping is a method of organizing materials developed by Novak and based upon the theoretical work of Ausubel. This technique had been shown to produce superior results on cognitive testing of students who had used the method rather than conventional outlining procedures.

Merrill divided a group of preservice elementary teachers into high- and low-achieving groups, tested them with an attitude toward mathematics scale, and taught them to concept map. All students were asked to map the division concept.

After analyzing the division maps, Merrill reported that none of the preservice teachers could map the division concept at the 75 percent criterion level, although students of the high-achieving group were significantly better at concept mapping than the low-achieving group of students. Merrill recommended teaching the division concept to prospective teachers in a way that would enhance understanding and would curtail emphasis upon algorithmic work.

Trent (February, 1987) surveyed the fifty state departments of education and a random sample of colleges of education at state universities to determine the adequacy of junior high school mathematics teacher preparation, recommendations for inservice programs, and whether or not there was a shortage of well-trained junior high school mathematics teachers.

Trent concluded from an analysis of the survey results that: (a) junior high school mathematics teachers were inadequately prepared in either content or methods of teaching; (b) there was a shortage of well-trained junior high school teachers; (c) universities did not offer relevant curricula for junior high teachers; (d) few schools offered a master's degree in junior high school teaching; (e) most junior high teachers had a minor or less in mathematics; and (f) the methods courses for junior high teachers were inadequate.

Computers and Preservice Teachers. Investigations of computer use with preservice teachers were conducted by Henry and Holtan (November, 1987), Battista (April, 1987), Rucinski (March, 1987), and Krach (January, 1987).

Henry and Holtan randomly assigned 26 college students of two elementary mathematics methods courses to two groups. Both groups

were pretested and given one hour of lecture/demonstration on microcomputer operations, terminology, and classroom applications. Following this lecture, the first group was given a one-hour hands-on laboratory experience with microcomputers, whereas, the second group was given a one-hour, in-class demonstration covering the same materials as the first group had experienced in the laboratory situation. Finally, both groups were posttested.

Henry and Holtan concluded that two hours of instruction did significantly improve students' knowledge of microcomputers, but that there were no significant differences in the extent of the knowledge of the two groups. They concluded that intitial computer work with students should emphasize discussions and demonstrations rather than hands-on laboratory work.

The use of Logo with 69 preservice teachers enrolled in a special geometry course was studied by Battista. After about three weeks of instruction, each class was randomly divided into two groups. One group was given assignments which were to be completed using Logo to investigate various geometrical concepts. The other group completed equivalent assignments using paper, pencil, ruler, compass, and protractor. At the conclusion of this treatment period, the two groups switched treatments while studying a different geometric assignment.

Battista concluded from the study that using Logo with preservice elementary teachers was not as effective as using paper-and-pencil investigations. He offered several explanations for the Logo group's poor performance. First, students had had difficulty transferring their laboratory experiences with Logo to paper-and-pencil applications. Second, because of the shortage of microcomputers (two or three had to share a microcomputer) the Logo group may have had less actual manipulative experience than the paper-and-pencil group. The researchers concluded that a great deal more research on the effects of Logo to teach mathematics is needed in the future.

Rucinski investigated the possible effects computer programming would have upon developing problem-solving strategies in preservice elementary teachers. The experimental group were students enrolled in a course entitled "Microcomputers in Education," and the control groups were students studying either "Metric Geometry and Teaching Elementary Mathematics" or "Instructional Tasks in the Secondary School." The students of the experimental group were taught BASIC. The groups were pretested and posttested with special problem situations designed in the 1960's at the Loyola University Psychometric Laboratory.

Rucinski found that an introductory course in computer programming did not have a statistically significant effect upon the problem-

solving abilities of the students in this course. He suggested that different results might have been obtained if students whose majors were outside of elementary education had participated in the study.

Logo was also the focus of investigation by Krach. He compared two groups. One group composed of 54 elementary education students participated in an introductory Logo microcomputer laboratory and a traditional manipulative materials elementary probability laboratory experience. The second group of 47 elementary education students was exposed to the same Logo microcomputer laboratory as the first group, but were given experiences in an elementary microcomputer probability laboratory instead of the traditional manipulative probability laboratory. The groups were tested on probability concepts at the conclusion of the experimental period. On this test, the first group, those who had had a Logo laboratory in addition to traditional manipulative materials, outperformed the group having had the microcomputer probability laboratory in addition to the Logo laboratory.

Research on computer use with preservice teachers did not show any advantages for computers when compared with conventional instruction. The criteria used were improvement in problem-solving abilities or achievement. This finding is consistent with research on computer use at the pre-college level.

Student Teaching. Student teaching is very often the culmination of a teacher education program. It provides prospective teachers the opportunities to experience the "firing line" of teaching. Many prospective teachers regard these experiences as the most important phase of their professional training. Consequently, student teaching deserves much research attention.

In 1987, several studies related to student teaching in mathematics were reported. We shall review two of those studies: one completed by Smith (December, 1987) and another by Tooke (June, 1987).

Smith surveyed student teachers from seven Pennsylvania state universities to determine their perceptions of their mastery of the principles and techniques of diagnostic mathematics teaching. A modified version of the Diagnostic Instrument of Supervision was used in the survey. The data, which included the diagnostic

instrument results, analysis of written course descriptions, and reports of instructor interviews, were designed to help student teachers evaluate their teaching techniques and to provide information to the institutions for possible program modifications.

Smith reported that the results of the survey revealed that the student teachers recognized a need for improvement, but did not feel a need for improvement in any specific area of diagnostic teaching. The survey also indicated that a greater emphasis on the implementation of instructional techniques was needed in the teacher training programs.

Tooke determined correlations between selected areas of the mathematics training of secondary school student teachers and the achievements of their students. The student teachers taught their classes from a performance based unit approved by the classroom as well as the college supervisor. The percentages of objectives attained by the students were the measures of achievement of the students that were correlated with elements of the student teachers' training.

The achievements of students in pre-algebra secondary mathematics courses correlated positively ($r = .63$) with the student teachers' grade point averages in college pre-calculus courses. The achievements of students in algebra and higher courses correlated positively ($r = .80$) with the grades of the student teachers' geometry course work. In general, the achievements of students correlated positively with the number of semester hours of college mathematics completed by the student teachers.

Research indicated that the achievements of students of student teachers correlated highly with the numbers of semester hours of college mathematics courses completed by the student teachers.

Inservice Teacher Education

We shall review research on inservice teacher education from two viewpoints. First, we shall look at research related to how teachers think about mathematics and the teaching of mathematics. Second, we shall review research related to the actual inservice education of teachers. Research on both of these topics was not extensive in 1987.

Teachers' Thinking About Mathematics and Teaching. We shall review four studies related to teachers' thoughts about mathematics and the teaching of mathematics. Schmidt, Porter, Floden, Freeman, and Schwille (September-October, 1987) identified four patterns of teacher content decision making, Putnam (Spring, 1987) reviewed live and simulated tutoring of addition involving first- and second-grade teachers in the San Francisco area, Russell (May, 1987) investigated the thinking of teachers about children and mathematics, and Thibodeau and Cebelius (February, 1987) studied the self-perceptions of special education teachers toward teaching mathematics.

Schmidt, Porter, Floden, Freeman, and Schwille had had extensive research experiences in studying factors that influenced teachers in curricular decision making on the individual classroom level. They had studied the influences of textbooks; objectives; standardized test results; discussions with principals, teachers, parents; subject-matter priorities; perceptions of student difficulties; and teacher subject matter enjoyment. In this study, they focused upon 18 teachers who were selected from suburban Michigan school districts. The 18 teachers of grades three to five were selected on the basis of their strong and distinctive views of mathematics. The school districts of the teachers were characterized as middle to upper middle class neighborhoods.

The 18 teachers were each interviewed for approximately two hours to probe their reasons for teaching mathematics, the kinds of lessons they had taught the previous year, and the strength of certain factors, such as parents, textbooks, other teachers, objectives, and tests, in their content decision-making. The interviews were tape-recorded, transcribed, and analyzed extensively by cluster analyses.

The analyses resulted in four clusters called "generic patterns of teacher content decision making." The four patterns were (a) the classic textbook follower (six teachers), who defined their content instruction as the first X pages of the textbook; (b) the textbook follower with strong student influence (six teachers), who used the textbook carefully but varied the page assignments for various groups of students in the class; (c) follower of conception and past experiences (three teachers), who were more influenced in content decisions by past experiences, the remainder of the curriculum, students, and other teachers than by merely the textbook; and (d) follower of district objectives (three teachers), who followed very closely the objectives of the district rather than merely the textbook.

Extensive interviews with primary grade teachers who had strong views about mathematics education revealed four generic patterns of curricular decision-making in mathematics instruction: (a) classic textbook follower, (b) textbook follower with strong student influence, (c) follower of district objectives, and (d) follower of conception and past experiences.

Putnam worked with four second-grade and two first-grade teachers from the San Francisco area. All were women with at least ten years of teaching experience. Each teacher tutored one second-grade student during two 20-minute sessions before or after school hours. These tutoring sessions were videotaped and were viewed by Putnam and the teacher, after each session.

After finishing two tutoring sessions with students, each of the teachers tutored six computer-simulated students in a session of about 2½ hours. In the simulation, a computer presented a series of students who make various systematic errors in addition exercises. The teacher interacted with the simulated student by selecting from a set of predetermined instructional moves. Each move was written on a card with an identifying number, which the teacher typed into the computer. The teacher continued this process until satisfied that the student had mastered the addition algorithm. At this point, the teacher was presented a retroactive prediction task. The teacher was given four addition exercises and asked to complete them as the student would have done before instruction began.

Putnam hypothesized a "curriculum script" model after analyzing the live and simulated tutoring episodes. He felt that teachers, instead of using a diagnostic model for tutoring, tended to follow a series of predetermined exercises as a substitute for the diagnostic model. Putnam named these predetermined exercise sets "a curriculum script."

The central finding of the study was that experienced teachers did not attempt to construct detailed models of their students' knowledge before the tutoring sessions. Rather, each teacher appeared to employ a curriculum script. The question was posed why teachers followed such "scripts" rather than attempt to diagnose individual student difficulties. It appeared that efficiency of instruction gained through years of classroom experience dominated the tutoring rather than an inefficient individual diagnosis. The curriculum script gained from past experiences was probably regarded

by teachers as the most efficient way to tutor as well as to instruct large numbers in the classroom.

Russell developed a taxonomy of elementary teachers' thinking about children and mathematics. Russell conducted an inservice seminar of 12 elementary teachers who discussed issues raised by engaging in problem solving, viewing videotapes of children solving mathematical problems, and sharing accounts of children's work from their own classrooms. The sessions were audiotaped and the teachers were also interviewed.

Russell developed a taxonomy of teachers' thinking that contained four major divisions: (a) mathematical learning and behavior, (b) teaching mathematics, (c) school and society, and (d) case descriptions. Russell felt that the taxonomy would be useful to other researchers interested in analyzing the thinking of elementary teachers about mathematics and children.

Thibodeau and Cebelius surveyed 141 special education teachers in three New England suburban school districts. The results of the survey indicated that (a) special education teachers did not discriminate between handicapped and nonhandicapped children regarding the importance of mathematics, (b) reading was regarded as more fundamental for children than mathematics, (c) more than 83 percent of the teachers perceived of themselves as being prepared to teach mathematics, (d) regardless of the number of years of experience, the majority of special education teachers were confident of their preparedness to teach mathematics, and (e) although perceived as being prepared, most felt a need for additional training in mathematics education in order to serve children better.

A survey of 141 special education teachers found that more than 83 percent of them felt prepared to teach mathematics to children, but most also felt a need for additional training in mathematics education.

Inservice Education of Teachers. Three studies dealt with the inservice education of teachers. Good and Grouws (June, 1987) developed a model for inservice training of teachers, Travers (January, 1987) studied conference participation of teachers, and Taylor-Ortega (December, 1987) focused upon the career patterns of mathematics teachers.

Good and Grouws observed that many mathematics teachers viewed mathematics as characterized by certainty and that they felt their function was one of helping students solve problems quickly and accurately. Good and Grouws felt that these viewpoints sustained, if not caused, poor student performance in mathematics found in national assessments.

Good and Grouws found from earlier work with elementary teachers that the developmental phase of instruction, which is designed to deepen understandings of skills, concepts, and other facets of mathematical learning, was often neglected by teachers. They identified five components important to development: (a) attending to prerequisites, (b) attending to relationships, (c) attending to representation, (d) generalizing concepts, and (e) attending to language.

They developed an inservice training program which involved teachers as professional partners. The program gave attention to the content of mathematics, teaching methods, and management issues. Each topic was approached from a dual viewpoint: a content strand and a management strand. The key topics in their program were problem solving, estimation, mental computation, and computers.

In the spring of 1985, Good and Grouws tried their inservice program with 16 fourth-grade teachers and seven principals of one school district. The plan consisted of ten half-day sessions every two weeks from December through February. The results of the inservice sessions were most favorably received by the teachers involved.

Travers surveyed 215 elementary, secondary, and postsecondary teachers of mathematics at a statewide conference of mathematics educators. It was concluded that different kinds of motivation existed for participation in the conference. Eighty-one percent of the participants indicated general indeterminate needs unrelated to specific roles. Eight percent indicated a specific problem-solving need for the conference, and seven percent felt needs which were based upon minor role changes.

Taylor-Ortega mailed questionnaires to 252 mathematics education graduates of Brigham Young University during the period of 1971-85. It was found that only 34 percent of the graduates were teaching mathematics in 1985. The most common pattern of employment was to teach for one to five years and then leave teaching because of marriage or a change of employment.

SUMMARY

In this section, we viewed teacher education from two broad perspectives: preservice teacher education and inservice teacher education. Although these two perspectives represent two distinct phases in the careers of teachers, there are common threads running through the two. For example, research revealed that some preservice teachers of mathematics were not pleased with the relevance of their undergraduate preparations in mathematics and pedagogy to their later careers. Inservice teachers frequently acted out "scripts" in the classroom when diagnoses and analyses of student difficulties were needed. The preservice education of teachers probably contributed heavily to this "scriptural" inservice behavior.

A study by Owens in which he intensively interviewed four preservice teachers showed that they viewed their undergraduate training with skepticism, questioning its relevance to their future careers. When teachers of 1967-69 were compared with teachers of 1983-85, it was found that the teachers of 1967-69 had studied fewer mathematics courses than teachers of 1983-85, but when their achievements on similar mathematical tests were compared, the 1967-69 teachers apparently knew more mathematics than their 1983-85 counterparts!

Although the junior high school has been a fixture in the American school system for decades, a survey of 50 states and numerous colleges of education revealed that few had special courses or degrees for junior high school teachers. Now that the middle school is competing for prominence with the junior high school, it seems clear that little will be done to further specialized training for junior high school mathematics teachers in the future.

The computer has entered teacher education, too. Researchers at this level as at the precollege levels were determined to study the by-products of working with computers. The usual hoped-for by-product, increased problem-solving skills, did not materialize for students using computers when compared with students not using computers. It seems abundantly clear that the goals of future researchers should be aimed toward finding the best and most efficient uses of computers as a tool to enhance instruction rather than searching for by-products.

In a study of student teachers, it was found that many did not employ a diagnostic attitude toward instruction, probably because they had not received adequate training in the diagnoses

of student difficulties. Not surprisingly, the achievements of students were correlated highly with the mathematical training of their student teachers, as revealed in a study by Tooke.

Researchers interested in inservice teachers found that, frequently, teachers were classic textbook followers, textbook followers subject to student influences, strict followers of district mathematical objectives, or followers of past experiences. Putnam found that elementary teachers were followers of a "curriculum script," that is, a prearranged plan to cover certain materials in a certain way. It appeared that the following of these scripts led teachers away from a more diagnostic model of teaching in which the teacher is guided in teaching by the difficulties that students experience. One wonders if the so-called "models" of teaching quite popular today, with their strict adherence to prescribed steps of teaching, lead teachers away from a diagnostic model of teaching. On the other hand, the many demands of the teachers may also mitigate against a diagnostic attitude in the classroom.

Research by Good and Grouws added to an understanding of the classroom rigidness of teachers. They found that many mathematics teachers viewed mathematics as characterized by certainty rather than a free way of thinking. With that mind frame, it is not surprising that many mathematics teachers felt that their primary responsibility was to help students solve problems rapidly and accurately. Good and Grouws had previously found that many mathematics teachers also neglected the developmental phase of mathematics teaching.

The research on teacher education reported in 1987 demonstrated that mathematics teacher education is not in the most healthy state. One hopes that the researchers of the future may find ways to improve the education of mathematics teachers.

V. COLLEGE-LEVEL INSTRUCTION

One usually associates "research" with a crucial function of higher education, and while most educational researchers are members of institutions of higher education, there has been considerably more research done at the pre-college than the college levels. Furthermore, there has been more research done related to the teaching of mathematics during the first two years of college than the last two years. In fact, one might observe that there appears to be an inverse relationship between the amount of research completed and the level of education that is researched.

In this section on college-level instruction, we shall begin with a discussion of research related to prominent researchers and teachers of mathematics at the college level. We shall consider research on content and learning including calculus, logic, proof, probability, and quadratic equations. Methods and learning, predictions of success in college mathematics, error analyses, and remediation will be discussed.

Topics that commanded considerable interest were computers, mathematical anxiety, and sex difference related to learning mathematics. Computers entered the college arena with much activity. Comparisons of computer-aided instruction with traditional instruction were done without much advantage being found for computers over traditional instruction. Mathematical anxiety has been a center of a great deal of research activity. The problem of anxiety is extremely complex, related to many other factors, including test anxiety. Because of the complexity of mathematical anxiety, the progress toward solutions has been limited. Workshops, special treatments, and special therapies have been tried. Frequently, the best solution is the "caring teacher." Research on sex differences continues to abound, and although differences between males and females were often documented, the basic reasons for those differences remained elusive.

Prominent Researchers and Teachers of Mathematics

Gustin (November, 1987) studied talented research mathematicians, and Johnson (July, 1987) focused upon prominent college teachers of mathematics.

Prominent Researchers. Gustin's study of talented research mathematicians was done in conjunction with the Development of Talent Project directed by Benjamin Bloom at The University of Chicago. It was hoped that the study would document the processes of exceptional cognitive development. The mathematicians chosen

for the study had demonstrated extraordinary achievements in mathematical research. They were interviewed to determine their home environments, learning experiences, and personal characteristics. Their parents, in most cases, were also interviewed to corroborate and extend the recollections.

The home environment was very important. The parents were typically well-educated professionals who prized academic success but did not pressure their children for particular career choices. The mathematicians were usually the oldest child in the family or the oldest male child. They were good students in high school who enjoyed working alone and excelled in science and mathematics. Usually, the mathematicians had not made a career choice when they began college. They enrolled in mathematics courses, excelled in them, and were studying graduate courses by their sophomore years in college. Following graduation, they enrolled in one of the better graduate schools. This was followed by ten years of intense work in their chosen specializations. They insisted that hard work was the essential ingredient for their success in mathematical research.

Prominent Teachers. Johnson studied teachers of mathematics chosen by the Mathematical Association of America for their reputations as fine teachers as well as fine mathematicians. The professors were filmed in action in the classroom, and eight films were chosen for special coding. The results of the coding revealed that: (a) the master teachers dealt with one major theme, with all other themes subordinate to that major theme; (b) the teacher usually gave an overview, followed by the body of the lecture, and concluded by a summary; (c) roughly half of the lectures were extensive and half were intensive; (d) the average number of variations and pauses was 6.8 per minute; and (e) almost all of the teachers included materials that fostered positive responses from the students.

Content Related to Learning

We shall consider research related to content and learning in three ways: (a) teaching calculus, (b) logic and proof, and (c) probability and quadratic equations. Calculus forms the backbone of most college departments of mathematics, because it is a course sequence required of so many different majors and fields.

Teaching Calculus. Rumore (July, 1987) investigated the teaching of the function concept to college students, Foley (March, 1987) studied the use of infinitesimals to introduce limits, Ferzola (June, 1987) traced the evaluation of the concept of differential and how that evolution influenced the teaching of differential in

a calculus course, and Moin (October, 1987) completed a meta-analysis of research related to four different methods of teaching calculus.

Rumore found no significant differences in teaching a unit on functions by presenting the formal definition followed by examples and a conventional method which incorporated the traditional approach with examples and nonexamples plus analyses of the examples.

Foley compared the introduction of the limit concept to community college students by the traditional epsilon-delta approach with the infinitesimal approach of a course in nonstandard analysis. An achievement test consisting of questions on computation of limits and comprehension of limits and continuity was administered to both groups. There were no differences found between the two groups on the test. It was found that males achieved higher than females, and those who had had exposure to calculus in high school responded better to the standard than the nonstandard approach.

Ferzola traced the evolution of the mathematical concept of differential from the time of Leibniz (1646-1716) to Cartan (1869-1951). In particular, he explored the evolution from Leibniz to Cauchy's formulation of the typical present-day textbook definition of differential. Ferzola also studied the evolution of the concept of differential in the calculus of functions of several real variables. He emphasized the changing meaning of differential in the contexts of the total differential, multiple integration, the generalized Stokes' Theorem, and the lemma of Poincare. Cartan's development of exterior differential forms was also explored. Ferzola provided a suggested pre-calculus and calculus sequence of topics based upon his study of the historical evolution.

Moin meta-analyzed the research on four techniques of calculus instruction: (a) self-paced mastery learning, (b) formative evaluation/feedback remediation, (c) computer-assisted instruction, and (d) strategy teaching. The self-paced techniques were similar to Keller's personalized system of learning, formative evaluation/feedback remediation used frequent testing for remediation of difficulties, computer-assisted instruction used the computer in various ways, and strategy teaching was based upon various innovative strategies designed to improve learning. The average effect sizes, which are the differences in means of the instruction and traditional instruction divided by the standard deviation, were 0.54, 0.29, 0.23, and 0.20, respectively. This would lead one to conclude that mastery learning was the most effective of the four techniques described.

Logic and Proof. Saulsbery (August, 1987) explored the development and application of Euler and Venn diagrams to the teaching of college logic, Hart (June, 1987) investigated the performances of

proof writing of college students studying elementary group theory, and Lewis (March, 1987) studied the perceptions of proof by university students and the relationships of these perceptions to achievement.

Saulsbery studied the logic diagrams of Euler and Venn along with selections of techniques used to teach logic with both diagrams. He analyzed and studied the assumptions and theory used by Euler in his geometric representations of logic. He also discussed the logic of Boole as a foundation for Venn's diagramatic approach.

Finally, Saulsbery compared the Euler and Venn methods, analyzing the merits, defects, and limitations of each, and the characteristics of each method to illustrate logical relations as well as the immediate inferences and moods of the categorical syllogism.

Hart described the performances of various classifications of students as they attempted to write proofs in elementary group theory. Twenty-nine college mathematics majors enrolled in three sequential abstract algebra courses were studied. The students were classified into four levels of understanding: (a) pre-understanding, (b) syntactic understanding, (c) semantic-concrete understanding, and (d) semantic-abstract understanding. The written proofs of the students on six problems were analyzed for correctness, processes used, and the students' assessments of the proof tasks. The major conclusion of the study was that the performances of the students could be traced to the stability or instability of their conceptual understanding of elementary group theory. Hart concluded that there are not experts and novices, but various gradations. The evolution from novice to expert is a rather unstable developmental process.

In describing mathematical understanding, there are not just "novices" and "experts" but rather many gradations between the two. The developmental process from "novice" to "expert" is an unstable one.

Lewis investigated the relationship of the understanding of proof and achievement of junior-level mathematics majors. Forty-seven students studying advanced calculus were given a questionnaire designed to measure various aspects of their understandings of proof. Using the results of the questionnaire, Lewis designed an interview script. The script was designed to assess subjective perceptions of

proof, the degree to which students enjoyed constructing proofs, and their confidence and ability to generate proofs. Achievement data were also collected, and no associations could be found between the script results and the achievement of the students.

Probability and Quadratic Equations. Knoth and Benassi (April, 1987) tested 598 students enrolled in an introductory psychology course on simple, additive, and multiplication probability problems, presumably, involving mutually exclusive or independent events. The test was designed to determine if students had knowledge of the fact that the conjunction of two or more events cannot have a probability greater than the probability of any one of the events. Eighty-three percent of the tested students failed to invoke this conjunctive concept.

Verheyden (November, 1987) compared an integrated presentation with a traditional presentation of quadratic functions with college algebra students. The integrated treatment emphasized graphs of quadratic functions and techniques for solving quadratic equations. The two groups were pre- and posttested. Verheyden found that the group instructed by the integrated method exhibited significantly better relational abilities, transfer ability (ability to solve higher degree equations than two), and ability to solve quadratic applications than the traditional group.

Methods and Learning

This section will be viewed from two broad perspectives: methods of learning and psychological factors. Methods of learning will include research on devices and techniques attempted at the college level to improve learning. Psychological factors will deal with the underlying reasons for certain approaches in college instruction.

Methods of Learning. Gonzales (January, 1987) studied diagram-drawing as a problem-solving strategy, Odafe (February, 1987) investigated problem solving with special students, Massey (April, 1987) looked into using algebra notecards by college students, Burton (January, 1987) focused upon essay writing in learning mathematics, Warden (September, 1987) studied test-like events with college students, Emley (July, 1987) investigated cooperative learning, and Ehlers (February, 1987) compared mastery learning and traditional lecture methods of instruction.

Gonzales compared two intact groups of college algebra students: one group was given instruction including diagram-drawing in solving problems and the other group only received instruction in algebraic techniques. Gonzales found positive but insignificant comparisons in favor of the diagram-drawing treatment in measures of learning

to solve verbal problems, recalling problem-solving instances, and solving problems which had not been taught earlier to the students.

Odafe compared problem-solving instruction with traditional lecture instruction with two groups of students from a special recruitment and admissions program. When comparing the two groups at the conclusion of instruction on course achievement with the skills section of a College Board examination and a researcher-constructed achievement test used as joint covariates, there were no differences between the groups. However, when only the researcher-constructed test was used as a covariate, the problem-solving group scored significantly greater than the lecture group.

Massey felt that using algebra notecards might help students become more efficient and effective learners of algebra. She experimented with traditional college students and adult learners (students who had graduated more than five years before beginning the algebra course) and found no significant differences between the two groups of students in their algebra achievements.

Burton had 50 students of an introductory level college mathematics course write essays about their homework and in-class mathematical activities. He compared their achievement test scores with a control group of 49 students over a four-week period and found that the essay writing did not affect achievement but did have positive effects on retention of knowledge.

Essay writing by college students about mathematical activities in a beginning-level college mathematics course had positive effects upon the students' mathematical retention.

Warden found that students who were given test-like items in lecture sessions did not achieve greater in a general education mathematics course than students who merely listened to the lectures. The material covered by the two groups was counting methods in an introductory probability course.

Remedial mathematics was taught in two ways to two groups of college-level remedial mathematics students: (a) the Team Assisted Individualization (TAI) Method, and (b) the Individualized Instruction (II) Method. Emley found no significant differences between the two methods on algebra achievement, but he did find that the TAI group had an 83 percent completion rate, whereas, the II group only

experienced a 54 percent rate. He also observed that introverts as determined by the Myers-Briggs Personality Indicator functioned more effectively in the TAI than the II settings.

Ehlers compared an "under 21" age group with a "21 and over" age group, while they were studying college algebra under two learning conditions: a modified mastery learning technique and a traditional lecture method. She found that: (a) students of the "under 21" age group achieved higher than the "21 and over" age group in the mastery learning groups, but that when comparing students in both lecture and mastery learning groups, the "21 and over" group achieved higher than the "under 21" group.

Psychological Factors. German (May, 1987) studied an analogy-induced schema in a new domain learning situation with college students, Alliger (February, 1987) investigated Piaget's binary combinations with students of a large Midwestern university, and Ponick (March, 1987) used animation on a microcomputer as a logical organizer for concept learning of 71 undergraduate students.

The use of advance organizers has been a popular topic of research since Ausubel's work in the 1960's. German constructed an analogy schema based upon the decimal number system for introducing binary, octal, and hexadecimal number systems. He presented the analogy instruction via computer to one group of students just before the advance organizer rules were given, and he presented the analogy to the other students immediately prior to the posttest. He found differences in favor of the earlier presentation in numbers of exercises completed, the time needed for the exercises, and in the time used to complete the computer module.

Alliger noted that Piaget and Inhelder had suggested that formal logical reasoning can be described by 16 binary combinations. Alliger wished to determine whether or not a paper-and-pencil test would be as effective as an individually administered test in determining the hierarchial order of the logic. Subjects for the study were students from mathematics classes in a large Midwestern university. An analysis of the paper-and-pencil test yielded hierarchial orderings of the 16 binary combinations similar to those found in the individually administered propositional logic game test. He concluded that the paper-and-pencil test could serve as a suitable substitute for the individually administered tests, thereby saving time and resources.

Ponick manipulated visual cognitive processes to determine their effects upon learning. Using the microcomputer as an instructional medium, Ponick compared four methods of presenting the sketching of families of mathematical curves. The four treatments were

defined by crossing two presentation variables (sequential versus simultaneous) and two selection variables (random versus guided). One of these groups also included animation.

Seventy-one university undergraduate students, non-mathematics majors or minors, were randomly assigned to the four treatment groups. Ponick found no significant differences among the four groups on the various measures of achievement utilized. However, there was a significant difference in the group receiving the animation treatment.

In using the microcomputer to present sketching of families of curves to college students, it was found that a presentation including animation was more effective than non-animated presentations.

Prediction of Success in College Mathematics

We shall consider prediction of success in college mathematics from two points-of-view: (a) various factors that affect performance, and (b) predictors of achievement.

Various Factors Affecting Performance. Sexton (September, 1987) investigated a self-efficacy-based model of mathematical performance, Witt (October, 1987) studied the effects of mathematics placement examinations, Grandy (October, 1987b) reviewed selection of mathematics as a college major by top-scoring SAT takers, Whitley (February, 1987) analyzed mathematics as a college filter to various majors, and Whitley (April, 1987) also studied various dimensions of college mathematical experiences as they affected choices of college majors.

Sexton had a set of college females take a series of mathematical word problem tests. In each of the tests, the students chose the levels of problem difficulty and how much effort to expend. A path analysis was used to probe the relationships.

Sexton determined that previous college mathematics courses and test scores were related to general mathematics self-efficacy. Past experiences were also major contributors to the levels of performance on the word problem tests.

Witt investigated the effects of mathematics placement tests upon students' grade point averages, success and completion rates

of mathematics courses. He studied the grade point averages, success, and completion rates for a period of semesters before the placement examinations were given and again four semesters following the administration of the examinations. He found that the grade point averages for students in intermediate algebra, trigonometry, and calculus were unaffected; however, increased completion rates of students in intermediate algebra and trigonometry were observed, after the placement examination had been given.

Grandy analyzed the trends in the selection of science, mathematics, or engineering as a major field of study among top-scoring SAT takers. Grandy found that the proportion of top-scoring SAT examinees majoring in science, mathematics, or engineering increased until 1982 and then decreased slightly. The percentage of top-scoring females planning to major in these fields also increased until 1982 and then decreased slightly. A similar pattern was observed for males.

Unfortunately for many students, their failure to study high school mathematics seriously limits their options to major in technical fields in college. Whitley (February, 1987) studied 745 freshmen at Arizona State University and observed that 45 percent of students majoring in scientific or technical fields had not studied trigonometry or fourth-year high school mathematics. Consequently, the majority of these students made very significant changes in their majors during their first year in college or dropped out of school.

In an additional report, Whitley (April, 1987) noted that 63 percent of the students made significant changes in their majors and 75 percent of these students dropped out of school. Mathematics grades provided a high index of frustration for many students. Some universities provide opportunities for students with deficiencies to reenter the scientific, technical fields. Perhaps, such opportunities will be limited in future years.

A study of the freshmen at a major university revealed that 45 percent of those intending to major in scientific or technical fields did not have trigonometry or fourth-year high school mathematics. A large proportion of those students changed majors or dropped out of college.

Predictors of Achievement. Landerman (December, 1987) studied attitudes and sex role orientations in predicting the mathematics achievements of college students, Johnson (September, 1987) focused upon the associations between testing strategies and performances in college algebra, Calhoun (October, 1987) investigated noncognitive variables in predicting performances of developmental mathematics students, and Goolsby and his colleagues (April, 1987) studied factors affecting mathematics achievements of high-risk college students.

Landerman analyzed pre-enrollment assessments of mathematics aptitude and various psychosocial variables to predict mathematics achievement and college major choices of 1052 college students. Regression analyses showed that regardless of gender, mathematics aptitude contributed most to the prediction of mathematics achievement, and attitudes toward mathematics contributed most to the prediction of college major choices.

College teachers use tests in their classes as a routine procedure and probably realize that such tests cause problems for students. Johnson studied 168 college algebra students and found that classes given frequent weekly quizzes, regular chapter tests, and required homework did not perform better on a final course examination than students in classes which only had a mid-semester examination in addition to the final examination. Attrition rates and attitudes improved with more frequent testing and homework.

Required homework and frequent testing appeared to improve retention of students in college algebra classes.

Calhoun compared 164 students enrolled in developmental mathematics courses with 170 students enrolled in non-developmental courses. He found that attitudinal variables (parental attitudes toward mathematics, confidence in learning mathematics, perceptions of the usefulness of mathematics, motivation, and the teacher's attitude toward the student as a learner) were related significantly toward the performances of the developmental mathematics students.

Goolsby and his colleagues studied students who were denied admission to a large state university because they did not meet admission standards but were subsequently enrolled in a special developmental algebra course. These students were regarded as "high-risk" enrollees. Using the students' course grades at the end of the term as a dependent variable, Goolsby found that certain affective variables (attitude toward success in mathematics, confidence in learning mathematics, perceptions of teacher's attitude, and mathematics anxiety) were good predictors of achievement in the algebra developmental course. A variable created by averaging the level of student confidence in learning mathematics and the mathematics anxiety score was the best predictor of mathematics performance.

A variable determined by averaging mathematics anxiety measures and measures of confidence in learning mathematics proved to be the best predictor of success of high-risk college students in a developmental algebra course.

Studies of Word Problems

Word problems have been the bane of the mathematical lives of many pre-college students. Consequently, it is not surprising that they also cause difficulties for college students. Word problems have been researched by several investigators in efforts reported in 1987. Whitehead (February, 1987) studied the relationships between successes of college students in solving word problems and evidences of metacognitive strategies, Reed (January, 1987) investigated a structure-mapping model for word problems with college students, Gerlach (January, 1987) focused upon individual schema development in solving word problems, Simon (March, 1987) looked into the use of diagram-drawing for the solution of algebra word problems, and Thompson (May, 1987) studied thought processes of community college students as they verbalized their experiences in solving algebra problems.

Whitehead attempted to determine if there were relationships between the successes of college algebra students in solving routine two-variable word problems and their use of metacognitive strategies in the problem-solving efforts. She defined two kinds of decision-making in solving problems: (a) tactical decisions and (b) strategic or metacognitive decisions. Tactical decisions included all algorithms and heuristics for solving a problem, and

strategic or metacognitive decisions included selecting a schema, deciding the directions that the solution should take, and deciding what to salvage if the solution plan failed.

Whitehead studied five students over a five day period and found a positive correlation between problem-solving test scores and scores of their use of metacognitive strategies determined during interviews with the students.

Research showed that the use of metacognitive strategies (selecting a schema, deciding directions for a solution plan, and deciding what to salvage from a failed plan) of college algebra students was positively related to their scores on a problem-solving test of two-variable word problems.

Reed hypothesized a structure-mapping model for word problems. The model was essentially a mapping of concepts and relations of one problem and the concepts and relations of another problem. A mapping is isomorphic if all the concepts and relations of one problem can be mapped onto all the concepts and relations of another problem. He defined "transparency" as a measure of how well students can match corresponding elements of two problems.

Reed conducted experiments with college students using four types of problems: (a) equivalent (same story, same solution procedure); (b) similar (same story, different solution procedure); and (c) isomorphic (different story, same solution procedure); and (d) unrelated (different story, different solution procedure).

Reed concluded from his experiments that students performed much better on isomorphic than non-isomorphic problems. Furthermore, the measure of transparency of a problem was a reasonable predictor of the abilities of students to notice isomorphs in two problems. Reed felt that structure-mapping models provided a sound theory for studying problem-solving behaviors of college students.

Gerlach rated word problems as X, Y, or Z according to increasing levels of difficulty. She then classified college students as novice (could only solve X), transitional (could solve X and Y), or expert (could solve X, Y, and Z).

Studying the novices, transitional students, and experts, Gerlach found that a larger proportion of students whose majors

were engineering and who had studied computer science were experts compared to other students enrolled in a beginning computer science course. She also noted that novices used less refined strategies than the other groups. Experts would verify their answers in more mathematical ways than other students.

Drawing diagrams is a problem-solving heuristic recommended by many problem solvers. Novice problem solvers often fail to draw diagrams. Simon interviewed remedial college mathematics students and found that the decision to draw a diagram in solving a problem depended upon (a) understanding the mathematics of the problem, (b) the diagram-drawing skills of the student, (c) conceptions of mathematics, (d) self-concept in mathematics, and (e) motivation to solve the problem.

Thompson observed students of a community college as they solved word problems in an introductory algebra course. He noted that the reading skills of the students as well as their mathematical skills were indicative of how well they could solve word problems. He also found that students had a higher rate of correctly solving problems if the problems were presented with a diagram that the student could relate to his or her own life.

Studies of Errors

In pre-college research, most error studies addressed computational errors of students. Error studies at the college level dealt more with the ramifications of dealing with errors than with the errors themselves. Kroll (February, 1987) studied difficulties related to solving inequalities, Geuther (January, 1987) focused upon error analyses in learning calculus, Neal (March, 1987) used errors to combat misconceptions of community college developmental students, Moldavan (June, 1987) examined the effects of warning intermediate algebra students about common errors, and Williams (Spring, 1987) investigated the effects of intentional teacher errors on achievements of college remedial mathematics students.

Kroll had 74 university calculus and precalculus students answer a questionnaire designed to determine their knowledge of properties of inequalities, their competencies in solving equations, and the difficulties they encountered in solving inequalities.

Fourteen students with high scores on the questionnaire were interviewed before and during their attempts to solve inequalities. Kroll found the difficulties associated with solving inequalities were due to confusion with the equality model and misinterpretations of logical connectives. He found that nearly half of the errors could be attributed to misunderstandings of the equality model. He

recommended that equations and inequalities be studied together and with the same intensity for each topic.

Geuther explored the nature of student difficulties in a first semester college calculus course. She collected achievement data and the results of a reflection attitude scale administered to all students. Her goal was to design grader materials that could be used by graders to diagnose student difficulties. The grader materials did appear to be useful in improving the diagnosis of student difficulties.

Neal collected lists of common errors found in the solution of linear equations by community college developmental mathematics students. One group of intermediate algebra students was taught in such a way that these common errors were emphasized and discussed. This group was compared with a control group which didn't hear the error emphases. The groups did not differ on a posttest given immediately following treatment, but the experimental group outperformed the control group on a delayed posttest.

In a similar kind of experiment to the Neal study, Moldavan identified 16 common errors made by intermediate algebra students. The errors fell into three categories: (a) errors due to defective algorithms, (b) errors due to incomplete algorithms, and (c) mechanical errors. Moldavan taught one class in which discussions of the errors were given, and he taught another class without such an emphasis. Unfortunately, no differences could be found between the performances of the classes, after the instructional phase was completed.

Williams taught two classes of remedial mathematics students in a two-year post high school technical program by two methods: (a) an errant-lecture method, and (b) the regular-lecture method.

In the Errant Lecture Group, Williams told the students that he would commit at least five errors during each class session. The students were requested to raise their hands if they detected an error being made. When an error was found, the error and the student's correction were compared. Consequently, the students were exposed to an example and also a non-example. If the error was not detected by the students, the teacher would pause and then correct the error himself. Using this procedure, Williams covered materials on factoring and fractions, linear and quadratic equations, graphs of trigonometric functions, exponents, radicals, complex numbers, exponential functions, logarithmic functions, ratio, and proportion.

When the Errant Lecture Group was compared to the Regular Lecture Group on examinations over courses content, no significant

differences were found between the two groups; however, the students of the Errant Lecture Group claimed to be more attentive in class than the non-error class.

Purposely committing teacher errors in class that could be corrected by students of a remedial mathematics group did not improve its achievement over a group in which the teacher did not commit such errors. However, students of the error class were more attentive than students in the non-error class!

Remediation of Mathematical Difficulties

Many colleges, particularly two-year institutions, offer remedial or developmental mathematics courses, that is, mathematics courses that are designed to correct deficiencies of incoming high school students. One may question whether or not this is a proper role of a higher education institution; nevertheless, such remedial or developmental mathematics courses appear to be permanent fixtures in many two-year colleges.

The discussion of research on remediation of mathematical difficulties will be viewed from two points of interest: (a) variations in courses that have been researched to improve the functions of such courses; and (b) studies of students electing developmental courses.

Variations in Remedial Courses. Researchers have looked at variations that may be made in remedial courses in hopes of improving their effectiveness. Goldberg (June, 1987) compared a course based upon a Gagné-Briggs design with a traditional course, Koch (July, 1987) investigated a course built upon multiplicative structure, Krinsky (September, 1987) experimented with self-paced versus instructor-paced courses, and Brown (March, 1987) studied the functions of a basic algebra course at a major university.

Goldberg designed a remedial college algebra course on the Gagné-Briggs theories. The special course was designed to develop students' abilities to categorize problem types and to associate these problem types through verbal cues with methods of solution. The problems were categorized by special factors, terms, and other characteristics. The verbal cues were precise labels which could be associated with the special characteristics. These were tied

to notes, solution flowcharts and computer-generated steps of the algorithms used in the solutions to the problems.

The course activities utilizing the special design featured in-class problem solving, self-grading, and discussions to provide immediate student feedback. When the students of the special course were compared with students of a traditional course on pre- and posttests of achievement, no differences could be found. The students seemed receptive and enthusiastic about the special course, in spite of its apparent lack of effectiveness when compared to traditional instruction.

Koch designed a course for college remedial mathematics around the concept of multiplicative structure, that is, a course in which the topics of multiplication, division, fractions, rational numbers, and functions were taught as an integrated unit. Koch designed a second course in which these topics were taught sequentially. When the results of the two courses were compared following the teaching of each to 53 students, no significant differences could be found in achievement measures. Correlational studies suggested that the students of the multiplicative structure course were better able to perceive relationships between rational number skills and proportional reasoning.

Krinsky compared a self-paced remedial mathematics course with an instructor-paced course that enrolled 485 students; 85 percent took the self-paced course and 15 percent took the regular lecture sessions. Krinsky found that the completion rates for the traditional lecture courses were greater than the rates for the self-paced courses. No differences could be found between the two groups in mathematics courses studied at later times.

At a major university, entering students who failed to achieve a cut-off score on a placement test were required to study a basic algebra course. It was found through a study by Brown that the special course did help students increase their knowledge of algebra, but that the course did not adequately prepare students for subsequent mathematics courses.

Students of Remedial Courses. Bassarear (January, 1987) investigated attitudes of college remedial mathematics students, Dinnel (May, 1987) studied differences between competent and less competent students, Porter (September, 1987) focused upon the achievements of remedial mathematics students, and de la Rocha (May, 1987) provided an ethnographic study of arithmetic in the everyday life of students.

Bassarear measured mathematical abilities, performances, and attitudes of students in a college remedial course. Questionnaires

were given in September and December, and 16 students doing very poorly were interviewed. Bassarear concluded that: (a) abilities and predicted grades provided the best predictors of success in the course; (b) ability was a stronger predictor for males than females, whereas, attitudes were a stronger predictor for females than males; (c) all students showed greater confidence in December than September; and (d) beliefs about mathematics did not change for any students.

Dinnel identified 32 competent mathematics students (scored in the highest quarter of the ACT Mathematics Examination) and 31 less competent students (scored in the lowest quarter of the ACT examination). The 63 students were given three cognitive processing tasks. Dinnel attributed differences between the competent and less competent students to abilities to access the external environment, to construct appropriate problem representations, and to plan solution strategies.

Porter identified 202 beginning university students who scored ten or eleven on the ACT Mathematics Examination and were required to study a developmental course before enrolling in college algebra, and 482 students who scored 12 or 13 on the ACT test and could enroll directly in college algebra. In analyzing the successes that these students experienced in the college algebra course, Porter found that the developmental course resulted in higher grades for its students, perhaps, providing justification for offering developmental studies.

Olivia de la Rocha conducted an ethnographic study of ten women associated with the adult Mathematics Project in California. She was interested in studying problem solving in every-day situations and chose dieting as an appropriate activity to study. Her findings suggested that conventional problem-solving techniques were inadequate to cope with the problem-solving required in the dieting activity.

Computers

The computer entered college mathematics instruction with a flurry of activity. Research related to computers can be partitioned into three sets: (a) studies dealing with computer use and student achievements, (b) studies that have varied the use of the computer in instruction, and (c) studies of programming and learning.

Computer Use and Student Achievement. Reid (February, 1987) compared three methods, including computer-assisted instruction, for teaching adult mathematics courses, Navarro (February, 1987) experimented with modes of guidance in computer-assisted instruction, Mitchell (August, 1987) did a meta-analysis of computer use and

student achievement, Gronberg (November, 1987) studied student achievement in solving systems of linear equations by computer, Meitler (July, 1987) also experimented with computer programs to learn solving of systems of equations, Isaacson (November, 1987) investigated the relationships between high school mathematics performance and success in college computer science courses, and Ganguli (February, 1987) studied the use of microcomputers in college intermediate algebra college courses.

Reid compared three methods for teaching mathematics to adult basic education students and General Educational Development students. The three methods were: (a) computer-assisted instruction using the PLATO system, (b) a tutorial method, and (c) a traditional teaching approach. No significant differences were found in the achievements of the three groups.

Navarro found, after an experiment with 30 college students, that they obtained higher achievement scores if they were able to use their preferred mode of guidance (student-guided or system-guided) in computer-assisted instruction on solving quadratic equations. In the student-guided mode, the subjects were able to move through the coursework by entering appropriate single letter commands, and they were free to choose computer graphics. Under the system-guided mode, the coursework prompted the students and guided them through the materials.

Mitchell conducted a meta-analysis of studies of innovative instructional methods utilized in lower division college mathematics courses. As is standard in meta-analytic studies, Mitchell examined and analyzed effect sizes for each study. Her analysis revealed that: (a) the ranks of methods in order of decreasing effectiveness were: tutoring, computer-assisted instruction, audio-tutorial instruction, individualized instruction, programmed instruction, laboratory and discovery methods, and television; (b) in terms of ability levels, the order of instructional modes were computer-assisted instruction and laboratory for high-ability students; computer-assisted instruction, individualized instruction, and programmed instruction for middle-ability students; and programmed instruction and audio-tutorial instruction for low-ability students.

Gronberg (November, 1987) tested three methods of instruction with college mathematics students: (a) traditional classroom instruction, (b) a computer program as an adjunct computer-assisted instructional method, and (c) the computer program as the primary instructional method. Twenty-three students were matched on the basis of current grades and assigned to each of the three groups. Three class hours were devoted to the experiment. Gronberg reported that the study supported the use of the computer in an adjunct instructional role.

Meitler (July, 1987) found no significant differences in the achievements of 327 students who were each taught by one of three methods: (a) a computer program which required the student to think of the next elementary row operation in the solution of systems of linear equations, (b) a computer program which allowed the student to select the next operation from a menu, and (c) a paper-and-pencil method of solving the system of equations.

Isaacson sent questionnaires to students who had completed the first computer science courses at three community colleges in Colorado and Wyoming. He determined Spearman rank correlations between the mathematics courses completed in high school and the course grade of the students. High school Algebra I and Algebra II grades were significantly correlated at the ten percent level of significance. If the top three grades (A, B, or C) were grouped and the remaining grades also grouped, then geometry and algebra II grades were significantly correlated.

Ganguli investigated whether or not the use of the microcomputer as a demonstration tool by the teacher made a difference in the achievements and attitudes of college students. The experiment was conducted with 118 college students enrolled in an intermediate algebra course. Two sections of students received instruction with the microcomputer as an aid and two sections received traditional instruction. The results of the study revealed that the experimental group achieved higher than the control group and also had more positive attitudes than the control group.

Varying the Use of Computers. Schimmel (May, 1987) studied the use of feedback by low-ability college students in an introductory algebra course, Kent (September, 1987) focused upon the effects of computer graphics on achievements in college algebra, Hawker (February, 1987) compared manual skills and computer algebra manipulations in calculus, Kiser (November, 1987) investigated whether or not computer visualization experiences would facilitate achievement, and Ritschdorff (March, 1987) developed some approaches for machine learning in mathematics education.

Schimmel studied low-ability college students taking a computer-based introductory algebra course. The students were permitted to select one of three types of feedback after each question of the program: (a) no feedback, (b) feedback providing the correct answer, and (c) feedback containing the answer and an explanation of how to determine the answer. The students' feedback selections were recorded by the computer.

Many students chose the same kind of feedback and individual questions. There were wide variations among the students following incorrect answers. Students who had chosen high information feedback more frequently than others had also shown significantly greater achievement in the course.

Kent tested two computer uses with 140 students studying college algebra at a liberal arts college. In both methods, the microcomputer was used in class as a teacher utility and in labs as a student utility. One group was exposed to lessons designed to guide students in the use of the computer and its graphic capabilities, and the other group used the microcomputer without the lessons. A slight difference was found in favor of the group that used the lessons.

Hawker found no significant differences in achievement between a group in which four calculus skills usually done manually were replaced by a computer algebra system and a group who did the manual manipulations. The experiment was limited to five homework assignments in a business calculus course.

Kiser experimented with two algebra classes at a Florida university. In one group the students received a highly visual computer treatment of linear programming for two weeks, and in the other group an overhead projector was used with the chalkboard to provide the most effective visual presentation without a computer.

Kiser found that students in the computer-enhanced instruction group did significantly better than students of the overhead projector group. Students of high spatial abilities did better in the computer-enhanced group than the traditional group.

Ritschdorff studied the possible applications and implications of research in machine learning to mathematics education. Similarities of machine learning to some aspects of mathematics education, particularly hierarchial ordering, specified preconditions, plausible reasoning, and the use of heuristics, were emphasized by Ritschdorff.

Computer Programming. Levin (October, 1987) investigated the relationships between method of teaching computer programming and improvement in problem-solving skills. Adner (January, 1987) studied an algebraic versus a programming approach in mathematical modeling.

Levin sought to determine if problem-solving skills of non-mathematically sophisticated college students were enhanced by learning to program in the BASIC Computer Language. Levin used two groups of students in the study. Both received the same instruction in BASIC syntax and decomposition of problems into

simpler problems. One group, the maximal guidance group, was required to submit a problem description and algorithm for each assignment. Both groups turned in a program listing with internal documentation. Levin found improvement in problem-solving skills for both groups and that students with low beginning skills received more benefit from maximal guidance than the traditional guidance.

Adner studied the mathematical modeling performance of students with different mathematical preparation levels. College students were divided into three groups according to their mathematical preparations, mathematical abilities (low, medium, high), and mathematical and programming backgrounds.

The students worked problems of three types: (a) those amenable to direct translation from English to mathematical symbolism, (b) those which required a "working backwards" technique, and (c) those requiring a specific strategy for solution. Two methods of solution were emphasized: (a) formulating an algebraic equation and (b) writing a computer program. Relatively few differences were found in problem solving among the various groupings of students except that for some problem types, differences were found in groups of differing mathematical preparation. Again, research seemed to demonstrate that one cannot find many differences due to variations in programming.

Research at the college level as well as research at pre-college levels demonstrated that the computer is a useful instructional tool, but that one cannot expect vast improvements in achievement because the computer is used. It seems clear from research that we should accept computers as very useful tools but not as a panacea for the ills of mathematics instruction.

Mathematical Anxiety and Sex Differences

We shall look at research in this section from three perspectives: (a) mathematical anxiety, (b) treatment of mathematical anxiety, and (c) sex differences.

Mathematical Anxiety. Hunsley (December, 1987) designed a study to determine the relations between test anxiety and math anxiety. He measured test anxiety with the Debilitating Anxiety

Scale of the Anxiety Achievement Test, and he measured math anxiety with the Mathematics Anxiety Rating Scale (MARS). The subjects of the study were 96 students in an undergraduate psychology course in a Canadian university. These students completed the test and math anxiety scales shortly after the beginning of the course. They also completed questionnaires designed to assess their anxieties, appraisals, and performance attributions before and after each of five examinations.

Hunsley found that math anxiety and test anxiety were significantly correlated. He also found that before examinations, students scoring high on test anxiety were more anxious, expected lower grades, and felt less well prepared than those scoring low. Elevated levels of math anxiety were associated with higher ratings of the importance of examinations. It appeared that scoring well on the test was very important to math anxious students. Surprisingly, importance of the examination to the student was not related to test anxiety. Some of the cognitive processes involved in math anxiety differed qualitatively from those found in test anxiety.

Although test anxiety and math anxiety were significantly correlated for college students, there were qualitative, cognitive differences between the two kinds of anxiety.

Hinkle (January, 1987) tested 75 students, 27 males and 48 females, on a series of personality, learning style, and math anxiety scales. She found that math anxiety was positively correlated with reflective observation and negatively correlated with concrete experiences. Math anxiety was also correlated with introversion and feeling. She found no sex differences in math anxiety. Hinkle recommended that mathematically anxious students should be taught by a "feeling" teacher.

Preston (January, 1987) investigated the prevalence of intensity of math anxiety in college students by sex and major, measured the stability of math anxiety over time, and studied its occurrence related to the backgrounds and experiences of students. There were 173 college students involved in the study.

Preston found that math anxiety was related to college majors, with technical students being somewhat less anxious; however, math anxiety showed little relationships to performance in mathematics. Math anxiety had a moderate relationship to mathematics background,

achievement, and avoidance of mathematics. It was found that the greater a student's level of math anxiety, the lower was his or her self-rating of mathematical ability.

Treatment of Mathematical Anxiety. Foley (February, 1987) described a math anxiety reduction workshop, Segeler (April, 1987) experimented with transactional analysis to reduce math anxiety, Gentry (October, 1987) compared two intervention methods to reduce the anxiety of female college students, and Lipsett (January, 1987) studied two instructional methods for reducing anxiety in remedial mathematics college students.

Foley described a special summer workshop at a major university designed to reduce math anxiety. One hundred thirty eight of 200 students who took a mathematics placement test scored less than 25 percent of the total score. The 138 students were invited to attend a workshop designed to reduce math anxiety; 75 chose not to attend and 63 attended. The 63 were pre- and posttested with the Fennema-Sherman Mathematics Anxiety Scale.

During the fall, the students enrolled in a basic mathematics course. Their achievement was measured by course final examinations constructed by the instructors of the courses. A greater percentage of workshop students earned A or B grades than did non-workshop students. Non-workshop students also had greater rates of attrition than the workshop participants.

Segeler tested entry level algebra students for math anxiety. Those having high math anxiety scores were invited to participate in a non-treatment control or a ten-week transactional analysis training workshop. The students were tested for math anxiety and completed attributional style questionnaires before and after the ten weeks of training.

Segeler found that the training group showed improvement on the scale designed to measure the usefulness of mathematics and on the composite scale of all mathematics attitudes. Overall, it was felt that the program did change attitudes toward mathematics and increased the likelihood that participants would find mathematics useful.

Gentry measured anxiety in two ways: (a) Sandman's Mathematics Attitude Inventory and (b) an electromyograph which measured skeletal muscle tension. Two interventions were compared: (a) cognitive restructuring (CR) and (b) modified progressive relaxation (MPR). The subjects of the study were 62 females enrolled in a private liberal arts college for women. Gentry found that CR led to significantly greater reductions in skeletal muscle tension than

MPR training. It was also found that CR was superior to MPR for students at advanced levels in mathematics.

Lipsett investigated the interactions between two methods of instruction (experiencing mathematics or direct expository) and two levels of mathematics anxiety (high or low). The subjects of the study were 160 students enrolled in a Puerto Rican university. Lipsett found no significant interactions between the treatment conditions and the anxiety levels. Lipsett concluded that either of the two instructional interventions could be used to improve the achievements of remedial mathematics college students.

Special math anxiety workshops, transactional analysis training sessions, cognitive restructuring techniques, modified progressive relaxation methods, and instructional intervention methods have been used to reduce the math anxiety of college students.

Sex Differences. Mura (January, 1987) reported on sex-related differences in the expectations of success in undergraduate mathematics, Ferrini-Mundy (March, 1987) discussed sex differences in achievements and visualization abilities of calculus students, and Elliott (March, 1987) studied traditional and nontraditional female and male college students.

Mura sent questionnaires to 1270 undergraduates majoring in mathematics or related fields in five Canadian universities. They were asked to predict their final grades in undergraduate courses. It was found that 51 percent of the women and 61 percent of the men overestimated their grades, 26 percent of the men and 26 percent of the women estimated their grades correctly, and 23 percent of the women and 13 percent of the men underestimated grades. These figures were statistically significant. Among mathematics majors, the percentages of overestimation, correct estimation, and underestimation were 57, 21, and 21, respectively.

No sex-related differences were reported in the confidence of students to complete their bachelor's degrees. However, women expressed less confidence than men in obtaining a Ph.D. degree. This was true of women specializing in mathematics as well as women in other fields. Fewer women than men expressed an interest in continuing to doctoral studies. When the levels of confidence were controlled, a smaller proportion of women than men expressed

a desire to engage in doctoral studies. Mura felt that a higher level of confidence was necessary for women to engage in doctoral studies than men. He urged further study to explain the cultural academic, and social mechanisms at operation in these situations.

A survey of five Canadian universities revealed that men tended to overestimate their grades in mathematics to a greater extent than women, were more willing to continue into Ph.D. studies in mathematics, and were more confident than women in their abilities to complete doctoral work.

Ferrini-Mundy selected a random sample of 334 students (167 men and 167 women) from 1054 students preregistered for a first-semester calculus class in a midsized university. Eighty-four of the students were eliminated from the study, so that 250 participated in one of two spatial training sessions or a control group. Analyses revealed no treatment effects upon calculus achievement or spatial visualization abilities. There were significant differences in favor of women for calculus achievement, and significant differences in favor of men in spatial visualization measures. Ferrini-Mundy felt that spatial training would benefit women in calculus to a greater extent than men.

Elliott measured causal attribution, confidence, perceived usefulness, and achievement of traditional and nontraditional college students. The examinees were enrolled in their first college mathematics course at seven campuses of the University of Maine. It was found that the affective variables were better predictors of mathematical achievement for nontraditional than traditional students, the achievements of traditional and non-traditional students were equally high, and there were greater differences between traditional and nontraditional students than between men and women on the same affective measures.

SUMMARY

In this broad section, we have discussed research related to college instruction in nine major categories: prominent teachers and researchers, content related to learning, methods and learning, prediction of success in college mathematics, studies of word problems, studies of errors, remediation of mathematical difficulties, computers, and mathematical anxiety and sex differences.

A group of studies examined factors related to the achievement/motivation factors in individuals judged as successful mathematicians or teachers of mathematics. Prominent researchers were outstanding students of mathematics from early in their careers. Their parents were supportive and prized academic success. Prominent teachers were well organized in their classroom presentations and encouraged interactions among students.

Comparisons of formal presentations of calculus with less formal methods didn't seem to reveal great advantage for either approach. In studying students attempting to write proofs, Hart found that the success of students was closely related to the stability of their understandings of the concepts of the subject. Hart observed that there were not merely novices and experts but rather many gradations between the two extremes.

Gonzales found positive effects for diagram-drawing as a heuristic in problem solving. Burton found that essay writing had positive effects in the college mathematics classroom, especially related to the retention of mathematical ideas. Ehlers observed that "21 and over" students demonstrated higher achievements in college algebra than the "under 21" age group. Perhaps, the added maturity of the older students and their more definite vocational goals influenced this difference.

Students are still arriving at institutions of higher education poorly prepared to cope with the studies necessary for technical careers. Whitley found that 45 percent of students intending to major in scientific or technical fields came to college studies without having studied trigonometry or a fourth year of mathematics in high school. Whitley also found that 63 percent of these students made significant changes in their majors after beginning their programs.

Word problems appear to cause as much difficulty for college students as for high school students. Drawing diagrams was found to be a useful heuristic for students, but novice problem solvers often failed to draw such diagrams. Furthermore, it was also found that students could better solve problems which were presented with diagrams than those that did not have diagrams.

One group of remedial mathematics students was taught in such a way that the instructor purposely made errors in class that were to be detected by the students. When the achievements of these students were compared with a class taught in the traditional way, no differences could be found. The students in the errant class were more attentive than the traditional group.

Remedial or developmental courses are common in two-year institutions. These courses are usually designed to provide

for deficiencies of students before they begin other programs. Self-paced instruction has frequently been compared with lecture methods. No clear-cut evidence was found to support either method.

Computers have become popular in college instruction. Numerous studies were done which compared computer-assisted instruction with traditional classroom instruction. As in pre-college research, the results were mixed, without a clear-cut advantage for either approach. Studies were also done to determine if experiences in computer programming would have beneficial side effects, such as, improved problem-solving capabilities of students. Unfortunately, results were again not decisive. The lesson, perhaps, to be learned from this research is that computers are powerful tools to enhance classroom instruction, but that their use will not insure miraculous learning outcomes. It seemed clear that future research should emphasize a search for better and novel ways to utilize computers rather than to dwell upon beneficial side effects.

Mathematics anxiety and test anxiety were found to be significantly correlated in studies by Hunsley. However, there were affective and cognitive factors that indicated the two kinds of anxiety were not the same. It appeared that math anxiety was related to choice of college major, with technical students being less anxious than non-technical students.

Treatment of mathematical anxiety has consisted of math anxiety reduction workshops, summer anxiety workshops, workshops involving transactional analysis, and special instructional methods. None of the these proved superior to any other, and all approaches enjoyed moderate amounts of success. Hinkle urged that care should be taken to insure that math anxious students are taught by a sympathetic, "feeling" teacher. Perhaps, that precaution may be the most effective treatment for most students.

Differences were found between males and females. Mura found, for example, that men overestimated their grades to a greater extent than women. Women tended to express less confidence in attaining higher levels, for example, the Ph.D. degree, than men. Men tended to outperform women in spatial visualization, and women outperformed men in certain aspects of calculus achievement. It seemed clear from these research efforts that differences between college men and women certainly exist, but the question that remained elusive is whether these differences are inherent or are the products of social and cultural factors. The latter hypothesis seems more reasonable but needs greater research efforts to justify its validity.

VI. RESEARCH SUMMARIES

Five types of research summaries will be reviewed in this section: (a) a comprehensive research listing, (b) Investigations in Mathematics Education (IME), (c) topical narrative reviews, (d) single topic reviews for practitioners, and (e) meta-analytic and best-evidence syntheses.

The comprehensive research listing is published each July in the Journal for Research in Mathematics Education. This massive compilation of research articles, dissertations, and summaries provides the mathematics education community an invaluable record of research completed each year, and it also provides the novice and experienced researcher a convenient starting point for research efforts.

Topical reviews, as the name implies, collect the research results related to a specific topic. Single topic reviews for practitioners is an effort of the National Council of Teachers of Mathematics through its journal, the Arithmetic Teacher. Each review dealt with a single topic and related research to useful ideas for school practitioners.

Meta-analytic and best-evidence syntheses represent efforts to introduce statistical rigor into the review process. Such reviews focus upon specific topics and statistically sum the results of many studies.

Comprehensive Research Listing

Each year Suydam (July, 1987) has complied an extensive listing of research in mathematics education for the previous calendar year. The seventeenth annual listing published in July, 1987, contained 27 research summaries, 210 research articles, and 392 dissertations listed alphabetically by author for research reported during 1986 for preschool through postsecondary school levels. Suydam searched 63 English-written journals varying from Academic Therapy through Vocational Aspect of Education for the 1987 report.

Suydam also provided short annotations for any reports in the listing which were especially relevant to mathematics education in North America. The separate listings of summaries, articles, and dissertations are followed by the names of the journals which were searched and an extensive index. The index is designed to assist

those readers who wish to locate references by a particular mathematical topic. In the index, the studies are listed under mathematical topics by author and under separate headings for articles and dissertations. The index does not include all reported studies nor is exhaustive for a particular mathematical topic. There were 18 topics in the 1987 index beginning with "achievement" and ending with "test analysis."

This comprehensive listing provides not only an invaluable record of research reported each year, but it also provides a most convenient beginning point for the experienced or novice researcher interested in mathematics education.

Investigations in Mathematics Education

Investigations in Mathematics Education (IME) is a quarterly publication of the SMEAC Information Reference Center with the cooperation of the ERIC Clearinghouse for Science, Mathematics and Environmental Education of The Ohio State University, Columbus, Ohio. Volume 20 of this journal was published as Winter, Spring, Summer, and Fall issues during 1987, under the editorship of Marilyn N. Suydam (Winter, Spring, Summer, Fall, 1987).

Each issue of IME contains about ten to twelve critical reviews of research reports that appeared in current publications. The reviews written by scholars in mathematics education are organized around five main headings: Purpose, Rationale, Research Design and Procedures, Finding, and Interpretations. A final section of each review, Abstractor's Comments, provides the reviewer an opportunity to comment freely about the study, its characteristics, its strengths, and its shortcomings.

Each issue of IME contains a nonannotated listing of mathematics education research studies reported in journals as indexed by Current Index to Journals in Education or as reported in Resources in Education. The listing also includes the EJ and ED numbers, respectively, for easy access to the ERIC database.

IME provides a valuable means for beginning or experienced researchers to review quickly a selection of current research studies along with evaluations of those studies by scholars in the field of mathematics education.

Topical Narrative Reviews

In this section, we shall consider topical reviews from three perspectives: (a) teaching, learning, and cooperative learning;

(b) skill learning, fractions, and mathematical creativity; and
(c) summer programs in mathematics instruction.

Teaching, Learning, and Cooperative Learning. Weaver (January, 1987) summarized briefly what research said about the learning of mathematics, Slavin (September, 1987) discussed behavioral and humanistic perspectives on cooperative learning, and Slavin (October, 1987) described a reconciliation of developmental and motivational perspectives on cooperative learning.

Weaver pointed out the confusion that exists among such terms as "curriculum," "instruction," "teaching," and "learning" of mathematics. In this report, Weaver focused upon the learning of mathematics and not upon research in teaching and instruction. He began by noting that Piaget's theory is a theory of intellectual development and not a theory of learning, teaching, instruction, or curriculum. Weaver further observed that research evidence suggests that the popular Piagetian tasks (e.g., conservation) are not useful readiness measures to identify children able to benefit from mathematics instruction. Research evidence indicated that some children who fail Piagetian tasks can and do learn mathematical concepts and skills hypothesized to require success on those Piagetian tasks.

Weaver also noted that information-processing theory replaced Piagetian theory as a broad explanatory model of cognitive development. Furthermore, information processing theory is often related to computer terms as a model for the mind.

Slavin (September, 1987) discussed behavioral and humanistic perspectives on cooperative learning by reviewing cooperative learning research that occurred in mathematical and other classrooms. In the behavioral viewpoint, cooperative learning rewards students on the basis of group achievements. The humanistic viewpoint focuses upon learnings that occur because of interactions of the members of the group with one another.

Slavin partitioned 55 studies into two groups: (a) 35 studies that based group rewards upon the sum of the individual learning of the members of the group, and (b) 20 studies that did not base rewards upon the sum of members' learning. In both sets of studies, cooperative learning methods were compared with control classes of conventional methods. In the first set of 35 studies, 30 favored cooperative learning over the control classes; in the second set of 20 studies, only three favored cooperative learning and two favored the control classes.

Of 35 studies of cooperative learning in mathematics and other classes, 30 studies favored cooperative learning in which the group was rewarded for the sum of its members' learning.

Slavin (October, 1987) noted that research on cooperative learning is motivated by two theories: (a) developmental theory, and (b) motivational theory. Developmental theory holds that interactions among students working on a common task creates cognitive conflicts that ultimately lead to a higher quality thinking. Motivational theory holds cooperative learning leads to efforts of group members to help and motivate one another. In the developmental viewpoint, group rewards are unnecessary; whereas, in the motivational theory, rewards are crucial. Slavin observed that more research on these two theories is needed to resolve many unanswered questions.

Skill Learning, Fractions, and Creativity. Ackerman (July, 1987) examined research on individual differences in skill learning. He considered skill learning of the types found in addition, mental multiplication, and memory scanning. The skills involved tasks with short times for solution, usually less than ten minutes per item and often much faster.

Ackerman recommended that future research on skill learning concentrate upon the distinctions between (a) subjects that commence and conclude skill-learning trials with superior performance, (b) subjects that begin and end with poor performances, and (c) the ability profiles of those subjects that begin with poor performance and demonstrate remarkable improvement. Ackerman suggested that an understanding of the relationships between skill learning and intelligence depends upon investigations of these three types.

Hope and Owens (Fall, 1987) analyzed research related to difficulties in learning fractions. They defined two problems that lead to difficulty in understanding fractions. First, since fractions involve two numbers, their understanding requires the simultaneous understanding of two unrelated ideas, and, second, the schools do not provide sufficient opportunities to deal with fractions in all appropriate settings.

Hope and Owens observed that children should learn how to partition continuous and discrete quantities before being introduced to fractions. Furthermore, they noted that researchers should address what work in fractions is actually needed in our calculator age. One possible solution is to concentrate upon decimal fractions and delay work on common fractions.

Hope and Owens concluded that:

> **"One of the most well established facts in all mathematics education literature is that performance on fractions is undesirably low."**
>
> **Hope and Owens, 1987, p. 37.**

Haylock (February, 1987) reviewed studies related to assessment of mathematical creativity in children. Haylock observed that the way children are taught and assessed in mathematics encourages children to think in narrow domains, rely on routine processes, and think convergently. He noted that in March, 1985, the ERIC database contained 4,732 articles and reports on creativity and only a handful of those were related to mathematics!

Haylock reviewed studies on creativity and school mathematics; mental sets, overcoming fixation, and rigidity; divergent production in mathematics; and divergent production tests in mathematics. He concluded that the ability to overcome fixations, both algorithmic and universe of content, in mathematical problem solving and the ability to think divergently in mathematical situations should be important components of any assessment strategy for mathematical creativity.

Summer Programs. Heyns (October, 1987) reviewed the literature on summer programs and found that they have been described as a means for enrichment or remedial instruction or as a technique for reducing cognitive losses during the summer among the disadvantaged. Summers are considered important, when the questions of time and cognitive inequality are crucial, but if the question is related to the amount of cognitive gain possible, summers are less important.

It is generally agreed that children learn at a slower rate during the summer than during the academic year. It also appears that summer programs are more effective to reduce declines in cognitive growth than to enhance such growth.

Single Topic Reviews for Practitioners

During 1987, the Arithmetic Teacher, published by the National Council of Teachers of Mathematics, reviewed research related to a single mathematical topic in each issue of the journal. The topics discussed by topic, author, and month of issue were: probability concepts in the elementary school, Shulte (January, 1987); calculators, Suydam (February, 1987); decimal fractions, Hiebert (March, 1987); problem solving, Bley (April, 1987); metacognition, Garafolo (May, 1987); addition, Fuson (September, 1987); rational numbers, Post and Cramer (October, 1987); zero and infinity, Wheeler (November, 1987); and microcomputers, Wilson (December, 1987).

The Topics. Shulte discussed research related to what elementary school students know and can learn about probability. He concluded that students in the elementary schools have some understanding of probability that increases with age and instruction and that some aspects of probability can be taught in the elementary schools.

Suydam considered research related to which topics calculators can be used to help students learn that topic. Such topics are: problem solving; number patterns and laws; counting; basic facts; and addition, subtraction, multiplication, and division with whole numbers and decimals. She also noted that handicapped children profit from calculator usage.

Regarding decimal fractions, Hiebert suggested that: (a) most students do not recognize that decimal fractions are another way of writing common fractions, (b) most errors are caused by students' confusion with memorized rules, and (c) many errors that students make with decimal fractions are not corrected as they advance in school. He recommended that more time be spent in developmental activities when decimal fractions are introduced, that rules should be developed from the meaning of symbols, and that students should estimate answers in decimal calculations before doing the calculations.

Bley recommended that direct instruction be used to help disabled students discover their skills, that teachers realize that often what disabled students actually see may be incorrect, and that disabled students process information more slowly than

normal students. She recommended the use of visual aids, cue words, and careful work with vocabulary meanings.

Garafolo reviewed research on metacognition and noted that students should become watchers, analyzers, assessors, and evaluators of their own mathematical knowledge and skills. He suggested that teachers ask students to think of everything they do in solving problems, what kind of errors they make, what do they do with difficult problems, and what kinds of problems are easiest or most difficult for them.

Fuson reviewed research related to adding by counting on with one-handed finger patterns. Children first find sums by counting the objects representing each of the addends. They soon learn that one can find the sum by counting on from the first addend. Fuson discussed some of the finger patterns that children may employ to find sums by counting on from one addend. Before teaching counting on as a method of addition, Fuson recommended that the children satisfy the following prerequisites: (a) understand the basic idea of addition, (b) perform addition by using objects to model sums less than ten, (c) possess a knowledge of small number sums, (d) read and write numerals to 20, (e) count objects to 20, and (f) count up to any word in a number-word sequence.

Post and Cramer considered research related to strategies of children in ordering rational numbers. They discussed strategies children used in the identical numerator situation (2/3, 2/5) and in the identical denominator situation (4/11, 7/11). They pointed out that words "more" and "greater" lead to confusion. For example, "more" can mean more pieces in a partitioned whole, or "more" can mean more area covered by each part. They cautioned that early fraction concepts should be introduced concretely to children so that they can better develop abstract representations of fractions at later times.

Wheeler traced the research related to children's understandings of zero and infinity from the 1930's to present times. The confusion between "nothing" and "zero" is discussed in addition to the difficulties associated with division and zero. Wheeler suggested some of the following practices to help students develop a better understanding of zero and infinity: (a) use zero in social contexts, e.g., street addresses and telephone numbers; (b) introduce infinity with such questions as, "Can you name a larger number?"; and (c) discuss infinity in geometrical as well as numerical settings, e.g., the number of lines of symmetry for a circle.

Wilson observed that research on microcomputer use in the elementary schools supported the following conclusions: (a) young children can program in BASIC or Logo, (b) open-ended software and programming can help students develop problem-solving abilities, (c) instruction supplemented by microcomputer drill-and-practice programs improves skills, (d) students using microcomputers develop positive attitudes toward mathematics, and (e) using the microcomputer is enhanced by teacher assistance. Wilson also suggested some group activities in which the microcomputer is used to solve problems.

Meta-Analytic and Best-Evidence Syntheses

Reviews of research fall into two basic categories: (a) narrative reviews, and (b) statistical reviews. Narrative reviews concentrate upon summarizing studies, analyzing their results, and providing a description of their meaning for educational practice. Statistical reviews may count the number of significant findings compared to the totality of findings or, as more recently, by meta-analyzing the results of studies. Meta-analyses concentrate upon determining effect sizes: the differences between experimental and control group means divided by the standard deviations of the control groups. These effect sizes are studied for homogeneity and related to study characteristics. The reviews described in the previous section were largely narrative; we shall now consider meta-analytic reviews and best-evidence syntheses. Best-evidence syntheses are reviews that attempt to combine the best features of narrative reviews and meta-analyses.

Meta-Analyses. Lewis (February, 1987) meta-analyzed studies on the effectiveness of kindergarten intervention programs, Sutawidjaja (September, 1987) meta-analyzed studies on the use of manipulative materials in early number instruction, Mitchell (August, 1987) conducted a meta-analysis upon studies comparing the effectiveness of innovative instructional methods in lower-division college mathematics courses, Moin (October, 1987) meta-analyzed studies comparing the relative effectiveness of various techniques used in calculus instruction, and Hembree (May, 1987) conducted a meta-analytic study of effects of noncontent variables on mathematics test performance.

Lewis studied 444 effect sizes calculated from 65 studies involving 3,194 kindergarten children. Strong positive effects were found on several variables related to school success including

intelligence, general academic ability, arithmetic skills, visual perception, and general information. Effect sizes for highly structured programs were greater than effects for less-structured programs, and long-term effects favored various levels of parental involvement. Lewis concluded that his meta-analysis supported widespread early intervention programs at the kindergarten level.

A meta-analysis of effects of kindergarten intervention programs favored widespread early intervention in kindergarten.

Sutawidjaja conducted a meta-analysis on 19 studies and 44 effects on the use of manipulative materials in teaching mathematics, primarily at the elementary level. Sutawidjaja found that the meta-analysis did not produce definitive, meta-analytic support for the use of manipulatives, and he recommended that more studies be completed in which treatments are carefully controlled. It should be noted that this meta-analysis, probably because of the lack of sufficient and rigorous studies, did not support contentions of Gruber in the 1840s, Brownell in the 1930s, and Dienes of today that the use of manipulatives enhances early mathematical instruction. The reader is also urged to read the Suydam and Higgins (1977) review on this topic.

Mitchell, as a result of her meta-analysis of studies on innovative instructional methods in lower-division mathematics courses, recommended variations in instructional methods, incorporation of specific innovative methods, and further investigations of innovative approaches with students of various abilities.

Moin's meta-analysis of various techniques of calculus instruction revealed the following effect sizes for the following techniques: self-paced methods (0.54), formative evaluation (0.29), computer-assisted instruction (0.23), and teaching strategies (0.20). Moin recommended that instructional programs in calculus incorporate a remedial program for students having difficulties.

Hembree (May, 1987) meta-analyzed 120 reports of research on the effects of noncontent variables on mathematics test performance. Effect sizes were determined for 18 variables. Conditions that

enhanced test performances were testwiseness training, praise, word problem pictures, and frequent testing. Conditions that lowered performances were the use of "none of these" as a multiple choice item and the presence of extraneous information in word problems.

Hembree recommended: (a) that there should be pretest discussions of the type of test, item arrangements, style of test, and instructions regarding guessing; (b) that students should be encouraged to draw diagrams for word problems; and (c) that teachers consider frequent testing.

Following a meta-analysis of studies of mathematics test performance, Hembree recommended that before tests are given teachers should discuss styles of tests, item arrangements, and types of questions possibly encountered; should encourage students to draw diagrams for word problems; and should test students frequently.

Best-Evidence Syntheses. Best-evidence syntheses combine features of narrative reviews with meta-analytic procedures. We shall consider two best-evidence syntheses: Slavin (Summer, 1987) and Slavin (Fall, 1987).

In the Summer, 1987, best-evidence synthesis, Slavin reviewed the literature on achievement effects of group-based mastery learning in elementary and secondary schools over periods of at least four weeks. Slavin described the literature search procedures, the criteria used for inclusion of studies (germaneness and methodological adequacy), and the computation of effect sizes. The review found essentially no evidence in favor of group-based mastery learning, if the criterion was based upon standardized achievement tests. If experimenter-made tests were used as a criterion, the effect sizes were positive but only moderate in size. Slavin recommended that more research should be done with long-term applications of mastery learning using broadly based achievement measures. He noted that mastery learning has made an important contribution to instructional methods.

> **"Mastery learning theory and research has made an important contribution to the study of instructional methods."**
>
> **Slavin, Summer, 1987, p. 208**

In the Fall, 1987, best-evidence synthesis, Slavin reviewed literature related to ability grouping and student achievement in elementary schools. He found that evidence did not support assignment of students to self-contained classrooms according to ability. Slavin did find that cross-grade assignments did increase achievement for selected students. Research supported cross-grade ability grouping for reading and within-class grouping for mathematics. It was also found that grouping was most effective when done for one or two subjects; otherwise, students should remain in heterogeneous classes. Slavin tentatively concluded, particularly for upper elementary schools that:

> **"Students should remain in heterogeneous classes at most times and be regrouped by ability only in subjects (e.g., reading and mathematics) in which reducing heterogeneity is particularly important."**
>
> **Slavin, Fall, 1987, p. 328**

SUMMARY

In this section, we have reviewed five categories of research summaries: (a) a comprehensive research listing, (b) Investigations in Mathematics Education, (c) topical narrative reviews, (d) single topic reviews for practitioners, and (e) meta-analytic and best-evidence syntheses. Each category of review made an important contribution to collecting research knowledge for the improvement of mathematics education.

The research compilations published each year for the past 17 years by Suydam are highly significant efforts to summarize and document the research efforts in mathematics education done during a full calendar year. The listing not only provides an important historical record but makes available in a single place research related to many topics.

Investigations in Mathematics Education, a quarterly journal, critically reviews about ten to twelve research studies in each issue. The reviews written by scholars in mathematics education summarize the studies and comment upon the strengths and shortcomings of each. IME also lists recent research studies reported in Current Index to Journals in Education and Resources in Education.

Topical narrative reviews were completed in 1987 on teaching, learning, cooperative learning, skill learning, fractions, mathematical creativity, and summer programs. Slavin concluded that cooperative learning in which the group is rewarded for the sum of its members' efforts is probably more effective than cooperative learning in which the group is not rewarded. Research on fractions continued to point out the many difficulties that students experience in learning fractions. Summer programs probably provide the best service for remediation and possible enrichment of students.

During 1987, the Arithmetic Teacher published each month a research review of a single topic. Such reviews varied from probability to calculators to metacognition. These reviews made available significant research results for use by practitioners.

One of the truly outstanding breakthroughs in research reviewing occurred with the introduction of meta-analyses. Meta-analysis represents a method for statistically summing the results of many studies on a single topic or topics. Best-evidence syntheses combine features of narrative reviews with meta-analyses. Meta-analyses were done on kindergarten intervention programs, manipulative materials in early number instruction, instructional methods in lower division college courses, calculus instruction, and noncontent variables related to performance on mathematics tests. Best-evidence syntheses were done on mastery learning and ability grouping.

VII. EPILOGUE: RECOMMENDATIONS FOR FUTURE RESEARCH

During the International Mathematical Congress in Paris in 1900, the great David Hilbert defined 23 mathematical problems whose solutions he felt were important to the development of mathematics in the years ahead. Hilbert must have felt that mathematical research needed direction so that it would not wander along unproductive paths. Hilbert was correct in his assessment because the progress of mathematical research has been measured in terms of the number of Hilbert problems that have been solved (Kramer, 1970, pp. 627-628).

Although there are many significant differences between mathematical research and research in mathematics education, it would seem that, from time to time, members of the field should stop and define the most significant problems that need investigation so that the field collectively does not wander along unproductive paths. Although the writer clearly recognized that he is not a David Hilbert and that the directions for the field should be the collective wisdom of the leaders of the field, he will nevertheless attempt to formulate some important problems that became apparent during the reading of hundreds of studies for this review of mathematics education research reported in 1987.

An Agenda For Research

In the next several pages, an attempt will be made to identify eleven problem areas whose study and investigation should be important to the advancement of the field of mathematics education during the years ahead. Problems will be defined in the broad areas of historical studies; problem solving; computers; early learning including constructivism, developmental activities, diagnostic activities, drill and practice, grouping practices, and team teaching; mathematical anxiety; gender differences; low achievers; textbooks; international studies; models of teaching; and evaluation. These areas are not mutually exclusive, so that there will be some overlap in their discussion.

1. Historical Study of Curriculum Development. If one reviews the history of the evolution of the mathematics curriculum since 1900, it appears that three forces have influenced its development. These three forces are psychological, sociological, and structural or disciplinary. From one period to the next, it seemed that one of these forces dominated to a greater extent than the other two combined. For example, the modern mathematics movement of the late 1950s and early 1960s seemed to have been dominated by concerns for the discipline of mathematics, the curriculum of the 1930s by sociological

aims, and the curriculum of the 1970s by psychological factors. This analysis is obviously an oversimplification of the forces affecting the mathematics curriculum. But what are the forces? Are those forces subject to change? Can they be changed?

Recently, the National Council of Teachers of Mathematics developed standards for school mathematics. Can these standards be implemented readily and efficiently? Without understanding the forces that shape the curriculum, the task may be difficult. Are there cultural factors in operation in this country that mitigate against change? It would seem that a careful study of the historical evolution of the mathematics curriculum done by bona fide "mathematical-historians," that is, researchers with scholarly attainments in both mathematics and history, would make a major contribution to the understanding of the forces influencing change in the mathematics curriculum.

2. Problem Solving. "Problem solving" has been the center of keen interest during the 1970s and 80s. It has been the subject of conferences, special workshops, institutes, and much research. Almost all mathematics educators would agree that the ability to solve problems is paramount among the skills that children should learn in school. Certainly "problem solving" has been the catchword of the 1970s and 80s.

If problem solving is so important, why isn't it evaluated on more standardized examinations? The obvious answer to that question is that problem solving is simply too difficult or, perhaps, impossible to evaluate by ordinary paper-and-pencil items. Are there other ways that problem solving could be tested? Could it be assessed using a new technology? If we wish to teach problem solving in the schools and have teachers adopt this as an important goal, then we must find ways to measure problem solving. If we continue to test only basic computational skills, then teachers will continue to emphasize computational skills and provide limited, lip service to problem solving.

3. Computers. It seems clear that we should call a moratorium on two kinds of research related to computers and think very carefully about a third kind of research. Much research at all levels, elementary through college, has focused upon comparing computer-assisted instruction or some computer-enhanced form of instruction with traditional lecture-type teaching. The results have not been clear. In some cases, the computer won the comparison race; in other cases, the computer lost. What does seem clear from this research is that the computer has not caused any damage to students.

The second type of research that has led to few useful conclusions is that searching for some side-effect or by-product

of computer use. Most often this research takes the form of attempting to measure problem-solving skills before and after students have engaged in a programming activity. The results are usually highly inconclusive.

It seems that we should abandon both kinds of research, that is, comparison research and search for by-products, in favor of studies that attempt to find ways to incorporate computers, calculators, and other technologies into the traditional classroom. These are marvelous tools and our efforts should be expended upon searching for the best ways to capture their values to enhance instruction.

The third kind of research that should be carefully reexamined is that research that models the mind after the computer. Reports of this research are filled with the jargon of information processing, words, such as, encoding, retrieving, long-term memory, short-term memory, chunks, traces: a rhetoric that seems to imply a profound theory. Is the model of the mind as a computer a useful and adequate analogue? Are there other models that would be far more useful? Is the mind more a radio receiver, a transmitter, or both? The problem with analogues is that they impose serious restrictions upon the original entity that the original entity may not at all possess. It may be that the mind as a computer is a highly restrictive model for research, especially, related to a subject as highly complex as mathematics.

4. Early Learning. Suggestions will be made in this section concerning constructivism, developmental activities, diagnostic teaching, drill and practice, error analysis, grouping practices, and team teaching.

a. Constructivism. Constructivism is a topic of considerable research activity. The idea of children constructing their own knowledge of mathematics is intriguing. Studies that describe children's attempts to develop algorithms for computation are fascinating. One cannot be less than impressed with the creativity of children and the novelty of their thinking. Most mathematics educators probably feel in their bones that these children are developing foundations for later mathematical development. But are they? It seems implicit in these research efforts that children constructing their own knowledge is better than meaningful instruction followed by drill and practice. But is it? Surely, if we expect teachers to adopt a constructivist's viewpoint then they must be provided evidence that this is a profitable mode when compared with meaningful instruction followed by drill and practice.

b. Developmental Activities. Research seems to indicate that elementary school teachers frequently neglect developmental activities, that is, those activities that attempt to provide the foundations for later understandings of a concept or principle of mathematics. Past research efforts indicated that developmental activities are a crucial ingredient in the instructional process. Why do so many elementary teachers neglect developmental work with students? Do they lack the mathematical backgrounds to provide this ingredient? Do the textbooks that they use neglect developmental activities? Would mathematics specialists in the elementary grades provide such activities to a greater extent than generalists? Many mathematics educators would probably hypothesize that the lack of mathematics specialists in the elementary grades is the primary reason. But is it?

c. Diagnostic Activities. Research has indicated that teachers often adopt "scripts" for teaching episodes, when a diagnostic, analytical approach was needed. Scripts are prearranged patterns of teaching which teachers have employed in the past and will probably use in the future. The problem with scripts is that they may completely ignore the difficulties that students are experiencing at a particular time. It is probably true that scripts are used in higher education to an even greater extent than pre-college education. The story of the college professor with yellow, tattered lecture notes is well known.

Why do teachers resort to scripts, when a diagnostic, analytic mode is needed? Is it a matter of efficiency, or have teachers simply never developed a diagnostic, analytic mind set when it comes to teaching? It seems clear that mathematics education would be substantially improved if teachers were more concerned with analyzing student difficulties than merely "covering the book."

d. Studies of Errors. A significant body of research has grown about studies designed to define and count error patterns of students. This has been done extensively in the elementary years, to some extent with algebra in high school, and also with remedial mathematics courses at the college level. The emphasis of this research is usually upon an identification of errors and usually mechanical explanations as to why the errors occurred.

While this research has been fascinating and carefully done, it has often neglected an analysis of possible misconceptions that lead to particular error patterns. Now that many kinds of errors have been identified and described, it would be fruitful for researchers to select a particular error pattern and attempt to determine what kinds of misconceptions or misteaching led to that particular error. Such research would probably employ clinical methods and large samples of students to fix the cause or causes of the error patterns. Once these causes have been isolated, they could be altered in the curriculum.

e. Drill and Practice. With the current emphasis upon problem solving and discovery learning, the words "drill" and "practice" have become pejorative, particularly to researchers and leaders in mathematics education. Yet when one visits a classroom, particularly in the elementary schools, there is ample evidence that drill and practice are very much alive in the schools. Do teachers have access to a conventional wisdom about drill and practice that has escaped so many mathematics educators? Do teachers employ drill and practice because they have never been trained in other methods? One can only speculate at this stage; whereas, observations and interviews with teachers might reveal the real reasons. Some reasons will be associated with the content of proficiency and standardized tests. As long as tests stress short, quick, recall responses to sequences of isolated items, teachers will attempt to prepare their students for such exercises by employing drill and practice long after its real values may be exhausted.

f. Grouping Practices. In-class grouping of students for reading in the elementary grades is a commonly accepted practice. Stories about the "red birds" and "robins" are legion in the mythology that has grown about reading instruction in the elementary classroom. Yet, when it comes to mathematics instruction, teachers tend to resist grouping practices and prefer a lock-step approach for all students. The result of this approach is often bored students at the top of the ability levels and lost students at the bottom. It would seem that if grouping has been effective for reading instruction that it should also be useful for mathematics. Again, why aren't in-class grouping practices a commonly accepted method for mathematics instruction?

g. Team Teaching. The use of teacher aides, peer tutors, parent volunteers, and student volunteers is becoming more

and more common in schools. The interests of industries and business in the classroom are also growing. One cannot help but feel that this trend will have highly beneficial effects upon children's learning and the schools. Has enough research effort been expended in order to find the most efficient ways to use this supply of talent? Parental volunteers are useful for grading papers, but are there many other ways that they can be useful? It is probably true that creative teachers have utilized this pool of talent in highly creative ways, but their experiences should be tested and disseminated to the mathematics education community.

5. Mathematical Anxiety. "Math anxiety" is a commonly used term today. Research efforts have been expended upon identifying levels of mathematical anxiety in various classifications of students and developing methods for treating mathematical anxiety. Research seems to indicate that it is related to "test anxiety" but yet separate from test anxiety. While these research efforts have been productive and laudable and probably should continue in the future, one cannot help but wonder if the role of the "caring teacher" and maybe the "uncaring teacher" have been investigated sufficiently. Invariably, when students are asked to identify factors contributing to their successes, often the elementary or high school teacher is identified as having made a significant contribution. Since the teacher is so important to the life of the student, what is the part that the teacher plays or does not play in creating student mathematical anxiety? Greater research efforts in identifying the characteristics of the truly "caring teacher" might pay big dividends toward the eradication of mathematical anxiety.

6. Gender Differences. Research has documented that there are differences between males and females. For example, it has been hypothesized and partially verified that females with high mathematical skills also have high verbal skills, whereas, males may have high mathematical skills with low verbal skills. Females tend to excel in mathematical tasks requiring precision of efforts; males tend to have greater spatial abilities. Of course, great variances in all of these skills are found, if males and females are studied independently. Obviously, they are highly capable women and highly capable men and lesser gradations of talent in each of the sexes.

The documentation of these differences is evident, but the question as to why these differences are observed is still clouded. Are there inherent differences in males and females or are the primary reasons cultural and social? The answer to those questions

would play an important part in designing educational opportunities for girls and boys that might compensate for inherent, social, psychological, or cultural differences in males and females.

7. Low Achievers. One of the scandals of mathematics education is the manner in which low achievers have been treated. From the early grades, the low achiever is reinforced with the idea that he or she is poor in mathematics. As a consequence of this, students often reach high school unable to study algebra. Because these students cannot negotiate the conventional curriculum, they are frequently "warehoused," that is, placed in classes where repetitive performance of computational skills is offered as the only academic menu. The result is a student who has not learned to cope with simple mathematical problem solving required in purposeful occupations. Often these students continue into remedial college courses where they may gain the needed skills.

Research related to low achievers could proceed in several directions. Language is one area that could profit from more study. For example, it has been shown that some oriental languages have greater congruence with number than other languages, but by the same token could the language of some low achievers lack a congruence with mathematical concepts that makes learning difficult for them?

Greater efforts should also be expended upon developing courses that are truly geared to the low achiever. These courses do not have to be less mathematical nor must they be technically oriented, that is, mathematics for particular occupations, but they must be designed to capture the interest and motivate the low achiever.

The teacher of low achievers must also be special. They must tolerate difficult teaching assignments and often unruly youngsters. They must believe in laboratory, hands-on learning in mathematics and possess the patience to perform under trying conditions. The price of success is well worth the efforts for these teachers.

8. Textbooks. In 1979, Begle wrote, "Any two textbooks differ on so many variables that it would be almost impossible to trace the specific variables which cause a specific difference, and without knowing which variables make a difference, we do not know where to start to improve textbooks" (Begle, p. 73). The research situation has not improved since Begle wrote that discouraging observation. Nevertheless, textbooks are important. Millions of dollars are spent each year buying them. The textbook remains the heart of the educational scene in many, many schools.

Since textbooks are so important, they deserve research attention. What features of textbooks are important? Does the

use of color enhance the learning process? Can textbooks be integrated with technologies? The answers to questions similar to these could, perhaps, save millions and possibly improve learning.

9. International Studies. Newspapers headline that the United States has lost the international race in mathematics and follow with discussions describing the ways in which students of the United States performed poorly in comparison to many other countries. Two basic questions should be answered in relation to these international contests: (a) Are the objectives tested by the international studies really important and crucial to American mathematics education? and (b) If the objectives are important, then how do we improve our students' performances so they can compete successfully?

Frequently, when teachers are asked these questions, they will respond that the objectives of international tests are not congruent to our objectives, and, even if they were, that social and cultural factors of other nations place them at an advantage over the United States. We may hear that the parents of Japanese children are more interested in education than parents of U.S. children. Is this really true? Are the cultural and social factors so strong that they overwhelm the effects of school efforts? It is doubtful that these questions can be answered in highly meaningful ways, but efforts should be expended to answer them, if we continue to place importance on the results of international studies.

10. Models of Teaching. Many school systems have adopted models for teaching. These models outline the steps that a teacher should pursue in presenting lessons in the classroom. While these models may help and guide teachers in organizing their presentations, they may not fit mathematics instruction. For example, some models discourage "bird-walking," that is, departures from the planned instruction. While routinely "getting off the subject" may not be generally desirable for teachers, there are numerous occasions in which the teacher may wish to depart from planned instruction in order to answer a student question, diagnose a student difficulty, or engage in group problem solving.

The solution to this dilemma is for mathematics educators to develop a model designed specifically for mathematics instruction that would permit departures from planned instruction for episodes of problem solving and diagnostic teaching. Such models would be welcomed by teachers and would be valuable to foster the distinctive characteristics of mathematics instruction.

11. Evaluation. Many teachers will complain, and rightfully so, that tests drive the curriculum rather than the curriculum

driving the test. Teaching for the test is all too common in American education. This attitude motivates extensive drill and practice on items that may appear on tests. Many tests deal with problem solving in superficial ways and emphasize routine computational skills. Calculators are often banned from testing sessions, largely because they would probably reduce the "validity" of the tests. Paper-and-pencil tests so critical in American education need a careful review as to their proper place in mathematics instruction.

We must find alternative methods of evaluation. Projects, notebooks, extensive problem-solving sessions, and writing about mathematics are a few alternatives that should be tried. A multifaceted evaluation model that would include these elements is needed and must be accepted by teachers, students, and the community in order to be effective as an evaluation strategy.

SUMMARY

In this epilogue, we have considered recommendations on eleven topics: historical studies; problem solving; computers; early learning including constructivism, developmental activities, diagnostic activities, studies of errors, drill and practice, grouping practices, and team teaching; mathematical anxiety; gender differences; low achievers; textbooks; international studies; models of teaching; and evaluation. Improvement in mathematics education is an important goal in American education, and it will require the collective efforts of all members of the educational enterprise including students, teachers, supervisors, administrators, parents, and researchers in mathematics education.

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